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1 Models for ordinal outcomes

1.1 Two-level ordinal analysis of TVSFP data

1.1.1 Introduction

The term "ordinal" is applied to variables where the response measure of interest is measured in a series of ordered categories. Examples of such variables include Likert scales and psychiatric ratings of severity. Nominal and ordinal outcome models can be seen as generalizations of the binary outcome model. The ordinal model becomes important when the outcome variable is not dichotomous, or not truly continuous. If an ordinal outcome is analyzed within a continuous model, such a model can yield predicted values outside the range of the ordinal variable. As with binary data, some transformation or link function becomes necessary to prevent this from happening. The continuous model can also yield correlated residuals and regressors when applied to ordinal outcomes because the continuous model does not take the ceiling and floor effects of the ordinal outcome into account. This can then result in biased estimates of regression coefficients, and is most critical when the ordinal variable in question is highly skewed. Armstrong & Sloan (1989) also report efficiency losses between 89% and 99% when comparing an ordinal to a continuous outcome, depending on the number of categories and distribution within the ordinal categories.

Extensive work on the development of methods for the analysis of ordinal response data has been undertaken by numerous researchers, including Hedeker & Gibbons (1994). These developments have focused on the extension of methods for dichotomous variables to ordinal response data, and have been mainly in terms of logistic and probit regression models. The proportional odds model proposed by McCulloch (1980) is a common choice for analysis of ordinal data. This model, which is described in detail in Section 1.1.3, is based on the logistic regression formulation.

In this chapter we will now build on the dichotomous model discussed earlier and introduce the ordinal model, illustrating the use of this model using the TVSP (Flay, *et. al.*, 1988) data previously used in this manual.

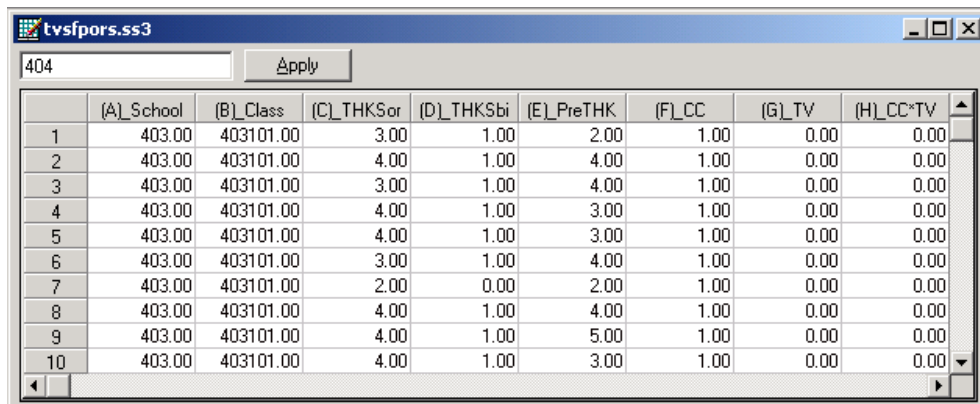
1.1.2 The data

The study was designed to test independent and combined effects of a school-based social-resistance curriculum and a television-based program in terms of tobacco use and cessation. The data from the study included a total of 1,600 students with both pre- and post-intervention scores from 135 classrooms drawn from 28 schools. Schools were randomized to one of four study conditions:

- A social-resistance classroom curriculum
- A media (television) intervention
- A social-resistance classroom curriculum combined with a mass-media intervention, and
- A no-treatment control group

A tobacco and health knowledge scale was used in classifying subjects as knowledgeable or not. In its original form, the student's score was defined as the number of correct answers to seven items on tobacco and health knowledge.

The structure of this study indicates a three-level hierarchical structure. However, for illustration purposes in this chapter we will consider a two-level structure in which students are nested within schools. Data for the first 10 participants on most of the variables used in this section are shown below in the form of a SuperMix spreadsheet file, named **tvspors.ss3**, located in the **Examples\Ordinal** subfolder.



	(A)_School	(B)_Class	(C)_THKSor	(D)_THKSbi	(E)_PreTHK	(F)_CC	(G)_TV	(H)_CC*TV
1	403.00	403101.00	3.00	1.00	2.00	1.00	0.00	0.00
2	403.00	403101.00	4.00	1.00	4.00	1.00	0.00	0.00
3	403.00	403101.00	3.00	1.00	4.00	1.00	0.00	0.00
4	403.00	403101.00	4.00	1.00	3.00	1.00	0.00	0.00
5	403.00	403101.00	4.00	1.00	3.00	1.00	0.00	0.00
6	403.00	403101.00	3.00	1.00	4.00	1.00	0.00	0.00
7	403.00	403101.00	2.00	0.00	2.00	1.00	0.00	0.00
8	403.00	403101.00	4.00	1.00	4.00	1.00	0.00	0.00
9	403.00	403101.00	4.00	1.00	5.00	1.00	0.00	0.00
10	403.00	403101.00	4.00	1.00	3.00	1.00	0.00	0.00

The variables of interest are:

- School indicates the school a student is from (28 schools in total).
- Class identifies the classroom (135 classrooms in total).
- THKSord represents the post-intervention tobacco and health knowledge scaled score, with 4 categories ranging between 1 and 4. The frequency distribution of the post-intervention THKS scores indicated that approximately half the students had scores of 2 or less, and half of 3 or greater. In terms of quartiles, four ordinal classifications were suggested corresponding to 0 – 1, 2, 3, and 4 – 7 correct responses.
- PreTHKS indicates a student's score prior to intervention, i.e. the number correct of 7 items.
- CC is a binary variable indicating whether a social-resistance classroom curriculum was introduced, where 0 indicates “no” and 1 “yes.”
- TV is an indicator variable for the use of media (television) intervention, with a “1” indicating the use of media intervention, and “0” the absence thereof.
- CC*TV was constructed by multiplying the variables TV and CC, and represents the CC by TV interaction.

In this chapter we will explore a random intercept model using the ordinal variable THKSord as outcome. In Chapter XXX, the post-intervention score was assumed to be a continuous variable. In contrast, here categories are created and the implied data collapse may lead to a loss of information and thus results may differ from those obtained previously.

1.1.2.1 Exploring the data

The focus in this chapter is on the influence of the intervention on the tobacco health knowledge scores of the students, as represented by the ordinal outcome variable THKSord. A cross-tabulation of the variables CC, TV, and THKSord are given in Table XXX1 below.

In general, students not exposed to the social-resistance classroom curriculum (CC = 0) seem to have less health knowledge than those students exposed to the social-resistance classroom curriculum (CC = 1), regardless of their exposure to media intervention. The opposite is true for students from groups assigned the social-resistance classroom curriculum (CC = 1).

Table XXX1: Crosstabulation of CC, TV and THKSord

TV			CC		Total
			0	1	
0	THKSord	1	117	62	179
		2	129	78	207
		3	89	106	195
		4	86	134	220
	Total	421	380	801	
1	THKSord	1	110	66	176
		2	105	86	191
		3	91	114	205
		4	110	117	227
	Total	416	383	799	

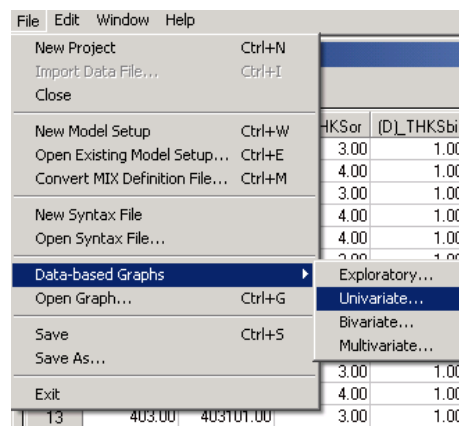
The trend is also apparent when the post-intervention scores are expressed as proportions (see Table XXX2).

Table XXX2: Observed proportion of high post-intervention scores

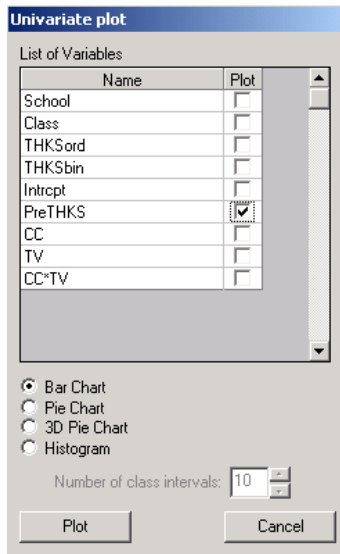
TV			CC		Total
			0	1	
0	THKSord	1	0.0731	0.0388	0.1119
		2	0.0806	0.0488	0.1294
		3	0.0556	0.0663	0.1219
		4	0.0538	0.0838	0.1375
	Total	0.2631	0.2375	0.5006	
1	THKSord	1	0.0688	0.0413	0.1100
		2	0.0656	0.0538	0.1194
		3	0.0569	0.0713	0.1281
		4	0.0688	0.0731	0.1419
	Total	0.2600	0.2394	0.4994	

First, notice that the outcome variable, THKSord, has a skewed distribution. By combining the proportions per category over interventions, we find that 0.2219 of the 1600 students had a value of 1 for THKSord, 0.2488 had a value of 2, 0.25 had a value of 3, and 0.2794 a value of 4 for THKSord. The monotonic increase in the proportion observed in each category of THKSord indicates that it would be inappropriate to try to fit a continuous model to the data.

The pre-intervention scores of the students may be used as a covariate in the analysis. To get some idea of the relationship between the scale score PreTHKS and the post-intervention score THKSord, an exploratory graph may be useful. To take a closer look at the distribution of PreTHKS, select the **Data-based Graphs, Univariate...** option from the **File** menu after opening the SuperMix spreadsheet **tvspors.ss3**.



The **Univariate plot** dialog box is activated. Select the variable PreTHKS, and request a **Bar Chart**. Click **Plot**.



The graph shown below is obtained. In contrast to the outcome variable THKSord, the distribution of the PreTHKS score has a lower mean, with very few students exhibiting extensive knowledge on the subject matter (PreTHKS = 5 or PreTHKS = 6).

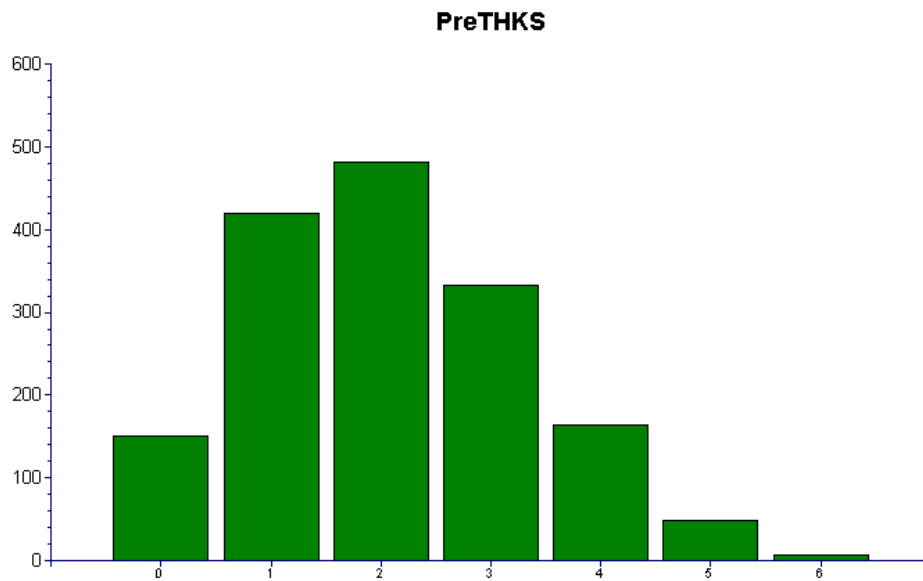
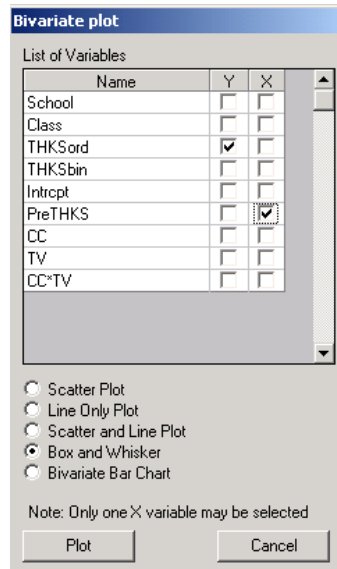


Figure XXX.1: Distribution of the PreTHKS scores

We now take a closer look at the distribution of the outcome variable at each distinct pre-intervention score value by utilizing the **Data-based Graphs, Bivariate** option on the **File** menu. By default, a bar chart will be produced. Select the variable THKSord in the **Y** column and the variable PreTHKS in the **X** column, and request a **Box and Whisker** plot before clicking the **Plot** button.



The figure below shows a reasonably steady increase in the mean THKSord with increasing PreTHKS scores. This seems to be expected: students with more initial knowledge ending up having higher post-intervention scores as well. Note that only 55 of the 1600 observations showed a score of 5 or higher on the pre-intervention score, and that no student obtained a post-intervention score of 7 out of 7.

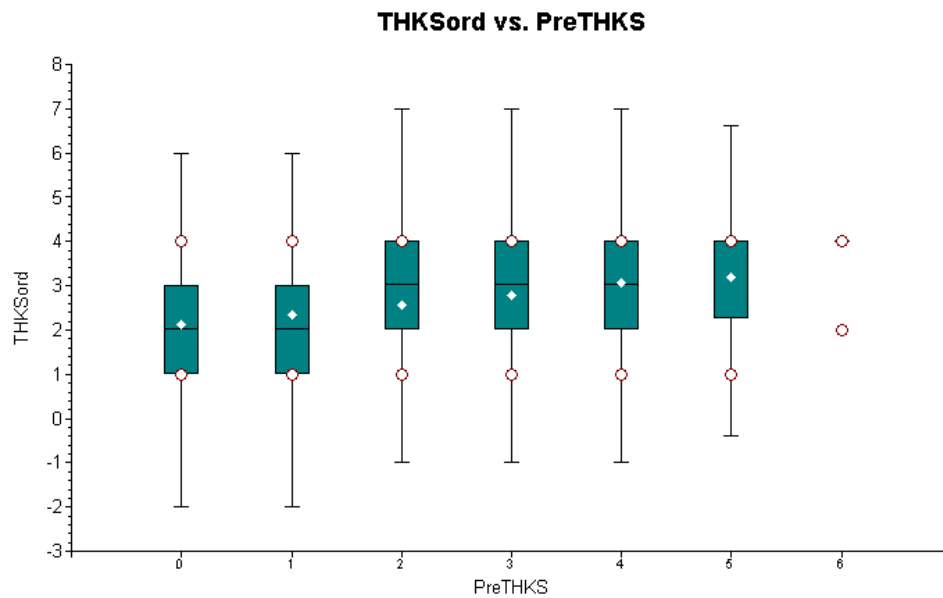


Figure XXX.2: Box-and-whisker plot of THKSord for values of PreTHKS

Finally, we also take a look at the mean pre-intervention scores of the students for each of the four subgroups. These are summarized in Table XXX3 below, and show that the mean pre-intervention scores do not differ much.

Table XXX3: Mean pre-intervention scores

Study condition	Mean
CC = 0, TV = 0	2.152
CC = 0, TV = 1	2.087
CC = 1, TV = 0	2.050
CC = 1, TV = 1	1.979

1.1.3 A multilevel ordinal model with logistic link function

1.1.3.1 The proportional odds model

The model we use for the analysis of ordinal data is based on McCulloch's (1980) proportional odds model, which characterizes the ordinal responses in C categories in terms of $C - 1$ cumulative category comparisons, specifically $C - 1$ cumulative logits. The McCulloch model can be written as

$$\log \left[\frac{P(y \leq c)}{1 - P(y \leq c)} \right] = \gamma_c - \mathbf{x}'\boldsymbol{\beta}$$

where

- $c = 1, \dots, C - 1$ for the C categories of the ordinal outcome
- \mathbf{x} is the vector of explanatory variables, plus the intercept
- γ_c represent the threshold parameter(s); and reflect the cumulative odds when $\mathbf{x} = 0$.

The positive association between a predictor variable x and the ordinal outcome variable y is reflected by β . It is assumed that the effect of x is the same for each of the cumulative odds ratios.

To illustrate, consider a model with a single predictor x . The odds that the response is less than or equal to c (for any fixed c) is divided by e^β for every unit change in x , as shown below:

$$\begin{aligned} \left[\frac{P(y \leq c)}{1 - P(y \leq c)} \right] &= \exp(\gamma_c - x\beta) \\ &= \frac{e^{\gamma_c}}{(e^\beta)^x}. \end{aligned}$$

On the other hand, the odds that the response is greater than or equal to c (again for a fixed c) is multiplied by e^β for every unit change in x :

$$\left[\frac{1 - P(y \leq c)}{P(y \leq c)} \right] = e^{-\gamma_c} \times (e^\beta)^x.$$

It can be illustrated that the ordinal model, when used for a dichotomous variable (coded 0 or 1), is equivalent to the model discussed in Chapter XX. In that model,

however, no thresholds were introduced. To motivate the ordinal regression model, it is often assumed that there is an unobservable latent variable (y^*) which is related to the actual response through the "threshold concept." An example of this is when respondents are asked to rate their agreement with a given statement using the categories "Disagree," "Neutral," "Agree." These three options leave no room for any other response, though one can argue that these are three possibilities along a continuous scale of agreement that would also make provision for "Strongly Agree" and "Disagree somewhat." The ordinal responses captured in y and the latent continuous variable y^* are linked through some fixed but unknown thresholds.

For the dichotomous model, one threshold value is assumed, while for the ordinal model, a series of threshold values $\gamma_0, \gamma_1, \gamma_2, \dots, \gamma_C$, where C equals the number of ordered categories, $\gamma_0 = -\infty$, and $\gamma_C = \infty$, is assumed. Here, a response occurs in category c ($Y = c$) if the latent response process y exceeds the threshold value γ_{c-1} , but does not exceed the threshold value γ_c . The cumulative probabilities are given in terms of the cumulative logits with $C-1$ strictly increasing model thresholds $\gamma_1, \gamma_2, \dots, \gamma_{C-1}$. In the current case, we will thus have $C-1 = 3$ cumulative probabilities, given in terms of 3 thresholds γ_1, γ_2 and γ_3 . The thresholds represent the marginal response probabilities in the C categories. We will illustrate the use of the logistic link function in this example.

To set the location of the latent variable, it is common to set a threshold to zero. Usually, the first of the threshold parameters (γ_1) is set to zero. Alternatively, the model intercept (β_0) is set to zero and $C-1$ thresholds are estimated.

1.1.3.2 The mixed-effect ordinal logistic regression model

A limitation of the model specified in the previous section is that it is assumed that the effect of covariates is the same across the cumulative logits. To overcome this limitation, an extension of the mixed-effects ordinal logistic regression model to allow for nonproportional odds for a set of regressors was developed by Hedeker & Mermelstein (1998). This generalization of the proportional odds model can be formulated as

$$\log \left[\frac{P(y \leq c)}{1 - P(y \leq c)} \right] = \gamma_c - [\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i].$$

In this model, as in the proportional odds model, the origin of the latent variable y is set by setting the first threshold, γ_1 , equal to zero. It is assumed that $\mathbf{v}_i \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_v)$. The unit of measurement is $\sigma = \pi / \sqrt{3}$.

For this model, the category probabilities are defined as

$$P(y_{ij} \leq c) = \psi(\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i))$$

and

$$P(y_{ij} = c) = \psi(\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)) - \psi(\gamma_{c-1} - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i))$$

where the cumulative standard logistic distribution function is

$$\psi(\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)) = \frac{1}{1 + \exp[-\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)]}.$$

Various link functions may be used with this model. If we define $G^{-1}[P(y_{ij} \leq c)]$ as

$$G^{-1}[P(y_{ij} \leq c)] = \gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i),$$

or, equivalently,

$$P(y_{ij} \leq c) = G[\gamma_c - (\mathbf{x}'_{ij} \boldsymbol{\beta} + \mathbf{z}'_{ij} \mathbf{v}_i)],$$

three types of models can easily be fitted:

- Using $G^{-1}(P) = \log\left[\frac{P}{1-P}\right]$ will give a cumulative logit model, *i.e.* a proportional odds model,
- using $G^{-1}(P) = \Phi^{-1}\left[\frac{P}{1-P}\right]$ will produce a cumulative probit model, and
- using $G^{-1}(P) = \log\left[-\log(1-P)\right]$, the so-called complementary log-log link, will give a proportional hazards model.

For more on the use of link functions, please see Section XXX.

1.1.3.3 A general multilevel ordinal model

The multilevel representation of the ordinal model follows the same logic as the dichotomous model discussed in Section XXX. When the multilevel model is expressed in terms of the observed response variable y , the level-1 model is written in terms of the cumulative logits, as shown below.

Level-1 model:

$$\log\left[\frac{P(y_{ij} \leq c)}{1 - P(y_{ij} \leq c)}\right] = \gamma_c - [\mathbf{x}'_{ij} \mathbf{b}_i].$$

where \mathbf{x}_{ij} represent the values of the covariates corresponding to level-1 unit j nested within level-2 unit i .

Level-2 model:

If all the elements of the coefficient vector \mathbf{b}_i are allowed to vary randomly across level-2 units, then

$$\mathbf{b}_i = \boldsymbol{\beta} + \mathbf{v}_i,$$

which models the level-2 effects as a function of an overall mean $\boldsymbol{\beta}$ and a unique random component $\mathbf{v}_i \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_v)$. The latter is also referred to as the level-2 residuals and indicates the extent to which a given level-2 unit differs from the average, as estimated by the first part of the level-2 model.

Note that the level-2 model does not depend on the response variable. As the regression coefficients $\beta_0, \beta_1, \beta_2$ and β_3 are without subscript, it is assumed that they do not vary across the categories and hence that the relationship between the predictor variables and the cumulative logits is not dependent on c . McCulloch (1980) referred to this as the assumption of identical odds ratios across the $C-1$ categories.

In practice, a subset of the coefficients \mathbf{b}_i are assumed to have fixed, but unknown, values. For example, a random intercept-and-slope model with 2 predictors of which the first has a random slope would have a level-2 model of the form

$$\begin{aligned} b_{0i} &= \beta_0 + v_{0i} \\ b_{1i} &= \beta_1 + v_{1i} \\ b_{2i} &= \beta_2 \end{aligned}$$

In this model, only the first two coefficients are assumed to vary randomly across the level-2 units.

Another characteristic of the current model is that a positive coefficient for a regressor indicates that the odds that the response is greater than or equal to c increases with an increase in regressor values. However, another formulation as shown below, in which the regression parameters $\boldsymbol{\beta}$ are identical but of opposite sign, is commonly used in survival analysis models (see Chapter XXX):

$$\log \left[\frac{P_{ijc}}{1 - P_{ijc}} \right] = \gamma_c + [\mathbf{x}_i' \mathbf{b}_i] \quad (c = 1, \dots, C - 1).$$

1.1.3.4 An ordinal model with 2 covariates and an interaction term

As in the case of the binary variable THKSbin, we intend to explore the relationship between the type of intervention, the pre-intervention scores of students and the ordinal outcome variable THKSord. We do so using a 2-level model, with students nested within schools.

Level-1 model:

At the first level, the pre-intervention score is used as predictor.

$$\log \left[\frac{P(\text{THKSord}_{ij} \leq c)}{1 - P(\text{THKSord}_{ij} \leq c)} \right] = \gamma_c - [b_{0i} + b_{1i} \text{PreTHKS}_{ij}] \quad (j = 1, \dots, n_i \text{ subjects})$$

Level-2 model:

At the school level, the types of intervention (represented by the dummy variables CC and TV) are used to explain differences in the intercepts of the groups. In addition, the interaction between CC and TV is included in the model.

$$b_{0i} = \beta_0 + \beta_2 \text{CC}_i + \beta_3 \text{TV}_i + \beta_4 (\text{CC} * \text{TV})_i + v_{0i} \quad (i = 1, \dots, N \text{ groups})$$

$$b_{1i} = \beta_1$$

It is assumed that $v_{0i} \sim NID(0, \sigma_v^2)$.

The model can also be formulated in a single expression as:

$$\begin{aligned} & \log \left[\frac{P(\text{THKSord}_{ij} \leq c)}{1 - P(\text{THKSord}_{ij} \leq c)} \right] \\ & = \gamma_c - [\beta_0 + \beta_1 \text{PreTHKS}_{ij} + \beta_2 \text{CC}_i + \beta_3 \text{TV}_i + \beta_4 (\text{CC} * \text{TV})_i + v_{0i}] \end{aligned}$$

Recall that the outcome variable has 4 categories. There are thus 3 thresholds. In this model

- $0 - \beta_0$ (remember that $\gamma_1 = 0$ for identification purposes) is the first logit (category 1 vs. categories 2 to 4) for groups with no intervention ($CC = TV = 0$). This logit is adjusted for the effect of PreTHKS.
- $\gamma_2 - \beta_0$ is the second logit, representing categories 1 and 2 vs. categories 3 and 4, for groups with no intervention ($CC = TV = 0$). This logit is also adjusted for the effect of PreTHKS.
- $\gamma_3 - \beta_0$ is the third logit, representing categories 1 to 3 vs. category 4, for the same groups and again adjusted for the effect of PreTHKS.
- The coefficient β_1 represents the effect of PreTHKS on THKSord.
- The coefficient β_2 denotes the PreTHKS adjusted logit differences between $CC = 1$ and $CC = 0$ (for $TV = 0$).
- The coefficient β_3 denotes the PreTHKS adjusted logit differences between $TV = \text{yes}$ and $TV = \text{no}$ (for $CC = 0$).
- The coefficient β_4 is the adjusted difference in logit attributable to interaction between CC and TV ($CC * TV$).
- The random school deviation is represented by v_{0i} . Note that we assume a single, fixed and thus common PreTHKS slope over the level-2 units.
- The interpretation of the coefficients is dependent on the coding of the variables used in the model.

1.1.3.5 Setting up the analysis

Using the data in **tvspors.ss3**, we consider the situation where students are nested within schools and fit a two-level model with the ordinal variable THKSord as outcome. We wish to examine the relationships between the outcome and the two intervention methods employed, simultaneously taking students' pre-intervention scores into account. To do so, we use the model described above with schools as the level-2 units.

Use the **File, Open Spreadsheet** option to activate the display of an **Open** dialog box. Browse for the file **tvspors.ss3** in the **Examples\Ordinal** folder. Select the file and click the **Open** button to return to the main SuperMix window, where the contents of the SuperMix system file are displayed. We are now ready to provide model

specifications.

We use the SuperMix interface to provide the model specifications. From the main menu bar, select the **File, New Model Setup** option. The **Configuration** tab of the **Model Setup** dialog box is displayed by default.

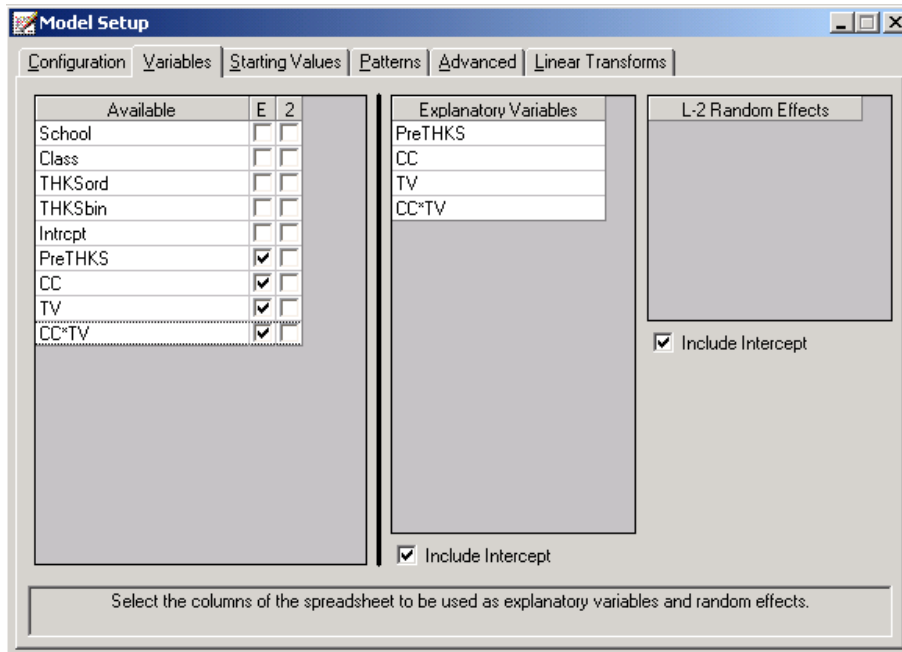
The screenshot shows the 'Configuration' tab of the 'Model Setup' dialog box. The 'Title 1' field contains 'TVSFP Ordinal' and 'Title 2' contains 'Students in Schools'. The 'Dependent Variable Type' is set to 'ordered'. The 'Dependent Variable' is 'THKSord'. The 'Categories' field is a table with 4 rows and 2 columns: 'Value' and an unlabeled column. The 'Level-2 IDs' is 'School'. The 'Level-3 IDs' is empty. 'Write Bayes Estimates' is 'no'. 'Convergence Criterion' is '0.0001'. 'Number of Iterations' is '100'. 'Missing Values Present' is 'false'. 'Perform Crosstabulation' is 'no'. A footer note says: 'Use the arrow keys or click on the desired tab to select the category of interest for the model.'

	Value
1	1
2	2
3	3
4	4

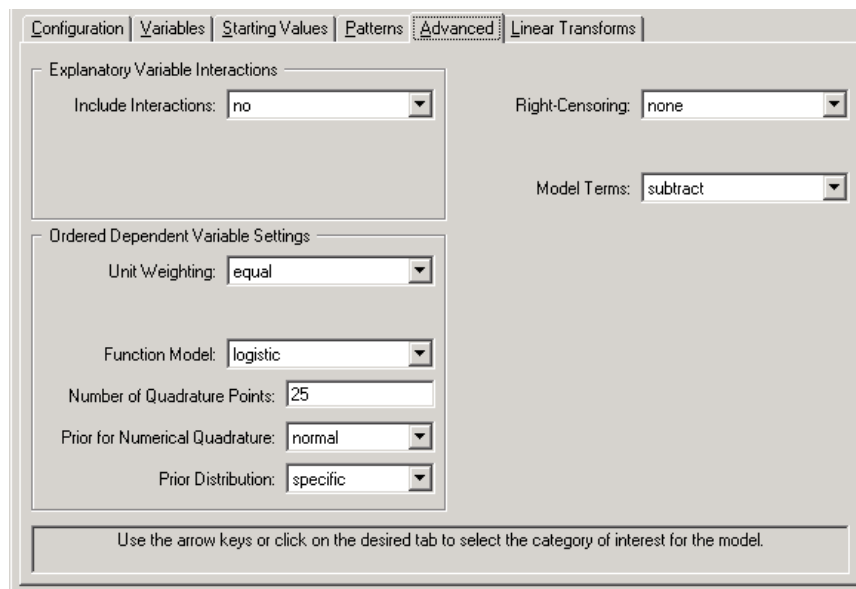
Start by selecting the ordinal outcome variable THKSord from the **Dependent Variable** drop-down list box. The type of outcome is specified as ordered using the drop-down list box in the **Dependent Variable Type** field. Once this selection is made, the **Categories** field is displayed. The School identification variable is used to define the hierarchical structure of the data, and is selected as the Level-2 ID from the **Level-2 IDs** drop-down list box. A title for the analysis is entered in the **Title** fields. A convergence criterion of 0.0001 is requested. By default, the maximum number of iterations allowed is 100. Default settings for all other options associated with this tab are used. Proceed to the **Variables** tab by clicking on this tab.

The **Variables** tab is used to specify the fixed and random effects to be included in the model. Start by selecting the explanatory (fixed) variables using the drop-down list box next to the first row in the **Explanatory Variables** box. After selecting all the explanatory variables, the random effect(s) at level 2 must be selected. In this case, we wish to allow only the intercept to vary randomly over the schools. By default,

the intercept is assumed to vary randomly over higher levels of the hierarchy as indicated by the checked boxes for the **Include Intercept** options.



We opt to increase the number of quadrature points to be used during estimation. To do so, select the **Advanced** tab and change the **Number of Quadrature Points** field to 25. We also request the use of a **logistic link** function from the **Function model** drop-down list box..



Before running the analysis, the model specifications have to be saved. Select the **File, Save** option, and provide a name for the model specification file, for example **TVOS.mum**. Run the analysis by selection the **Run** option from the **Analysis** menu.

1.1.3.6 Discussion of results

Portions of the output file **TVOS.out** are shown below.

Program information and syntax

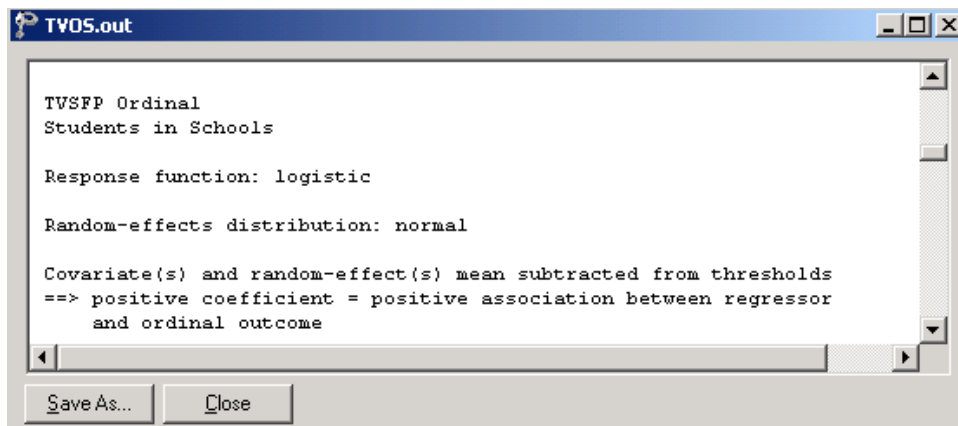
```

Model=Ordinal;
Options Converge=0.0001 Maxiter=100 Bayes=No ModelTerms=subtract
      NQuadPTS=25 Prior=normal;
Link=logistic;
Varnames= School Class THKSord THKSbin Intrcpt PreTHKS CC TV 'CC*TV' intcept;
Title=TVSFP Ordinal;
Title2=Students in Schools;
DataFile=C:\Program Files\SuperMix\TVOS.dat;
Level2ID= School;
Dependent= THKSord;
Categories= 1 2 3 4;
Predictors= intcept PreTHKS CC TV 'CC*TV';
L2Random= intcept;
FixPatType=Free;
Cov2PatType=Correlated;

```

At the top of the file, the syntax saved to the **TVOS.mum** file is shown. The first part states the selection of iteration control options, requests for Bayes residuals, and the specifications necessary to define the model fitted as an ordinal model with logistic link function. The second part of the syntax provides information on the structure of the data, the name and structure of the outcome variable, and the predictors included in the model. Note that this part now also includes information on the categories of the outcome variable and the link function selected.

The next section contains a description of the model specifications. The use of a logistic response function (logit link function), with the assumption of a normal distribution of random effects is indicated. It is also noted that covariate and random effect means are subtracted from the thresholds, implying that a positive coefficient indicates a positive association between the outcome and the predictor in question. To add the covariate and random effect means instead of using the default subtract setting, the **add** option must be selected in the **Model Terms** field on the **Advanced** tab of the **Model Setup** dialog box.



```
TVSFP Ordinal
Students in Schools

Response function: logistic

Random-effects distribution: normal

Covariate(s) and random-effect(s) mean subtracted from thresholds
==> positive coefficient = positive association between regressor
and ordinal outcome
```

The screenshot shows a window titled "TVOS.out" with a scrollable text area containing the following text: "TVSFP Ordinal", "Students in Schools", "Response function: logistic", "Random-effects distribution: normal", and "Covariate(s) and random-effect(s) mean subtracted from thresholds ==> positive coefficient = positive association between regressor and ordinal outcome". At the bottom of the window are "Save As..." and "Close" buttons.

Descriptive statistics and starting values

After the observation counts, descriptive statistics for all variables included in the model are followed by a frequency table for the categories of the outcome variable, and starting values for all coefficients. The starting values for the predictors CC, TV, CC*TV and PreTHKS are given in the first line (covariates), while the starting value for the variance component associated with the random level-2 intercept is given in the second line (var. terms). Finally, starting values for the second and third thresholds (γ_2 and γ_3) are given.

TVOS.out

Categories of the response variable THKSord

Category	Frequency	Proportion
1.00	355.00	0.22188
2.00	398.00	0.24875
3.00	400.00	0.25000
4.00	447.00	0.27938

Starting values

covariates	0.000	0.000	0.000	0.000	0.000
var. terms	0.574				
thresholds	1.137	2.202			

Save As... Close

Fixed effects results

TVOS.out

* Final Results - Maximum Marginal Likelihood Estimates *

Total Iterations = 39
 Quad Pts per Dim = 25
 Log Likelihood = -2119.743
 Deviance (-2logL) = 4239.486
 Ridge = 0.100

Variable	Estimate	Stand. Error	Z	p-value
intcept	0.08855	0.31306	0.28286	0.77728 (2)
PreTHKS	0.40331	0.04293	9.39513	0.00000 (2)
CC	0.92359	0.37128	2.48756	0.01286 (2)
TV	0.27485	0.31509	0.87231	0.38304 (2)
CC*TV	-0.46570	0.40613	-1.14667	0.25152 (2)

Save As... Close

The output describing the estimated parameters after convergence is shown next. Thirty-nine iterations were required to obtain convergence, using 25 quad points per dimension. The likelihood function value at convergence as well as the deviance are also given, and may be used to compare a set of nested models. The estimates are shown in the column with heading Estimate, and correspond to the coefficients $\beta_0, \beta_1, \dots, \beta_4$ in the model specification. Significant effects of PreTHKS and CC are observed. With the exception of the CC*TV interaction term, positive relationships between the predictors and the ordinal outcome variable are indicated by these results. We also note that the coefficient associated with the curriculum-based intervention (CC) is almost three times the size of the estimated coefficient for media intervention (TV).

Random effects results

The last part of the output file contains information on the random effects, thresholds, and calculation of the intraclass correlation coefficient. The variation in intercept over schools is estimated at 0.271, with the associated p -value of 0.001 indicating its statistical significance. The estimates of γ_2 and γ_3 are 1.242 and 2.420 respectively – recall that for identification purposes γ_1 was set to zero.

```

TV05.out
-----
Random effect variance term (standard deviation)
intcept          0.27118          0.09124          2.97204          0.00148 (1)

Thresholds (for identification: threshold 1 = 0)
  2              1.24187          0.08830          14.06354          0.00000 (1)
  3              2.42044          0.08361          28.94770          0.00000 (1)

note: (1) = 1-tailed p-value
      (2) = 2-tailed p-value

Calculation of the intraclass correlation
-----
residual variance = pi*pi / 3 (assumed)
cluster variance = (0.271 * 0.271) = 0.074

intraclass correlation = 0.074 / ( 0.074 + (pi*pi/3) ) = 0.022
  
```

Save As... Close

In the case of the fixed effects, a 2-tailed p -value is used, as the alternative hypothesis considered here is of the form $H_1 : \beta \neq 0$. As variances are constrained to be elements of the interval $[0, +\infty)$, the p -values used for these effects are 1-tailed. If the model is true, it is assumed that the level-1 error variance, σ_ϵ^2 , is equal to $\pi^2/3$ for the logistic link function, where π represents the constant 3.141592654 (see, e.g., Hedeker & Gibbons (2006), p. 157).

Finally, the calculation of the intracluster correlation is shown. In this calculation it is assumed that the residual variation, σ_ϵ^2 , is equal to $\pi^2/3$. The value of 0.022 indicates that almost all variation is attributable to students, rather than to the schools.

1.1.3.7 Interpreting the output

Model-based graphs

Activate the **Model Setup** window by clicking on it. Using the **Plot Equations for Outcome Variable** dialog box that appears when the **File, Model-based Graphs, Equations** option is selected, we can graphically depict the trend in post-intervention scores as a function of pre-intervention scores, taking the type of intervention into account. The dialog box below shows the selection of the predictor PreTHKS. Grouping of plots by the categories of CC is requested, while marking of the plots by TV is indicated by the selection in the **Mark** column. Two graphs will thus be displayed on the same set of axes: one for each value of the indicator variable TV. By default, all variables present in the model, but not selected for inclusion in the graph, will be assumed to have a value of 0. In the current situation, this means that CC*TV is kept constant at zero. In effect, the graphs are for students from schools where only one of the interventions was administered; students from schools where both were implemented would have a value of 1 on the variable CC*TV.

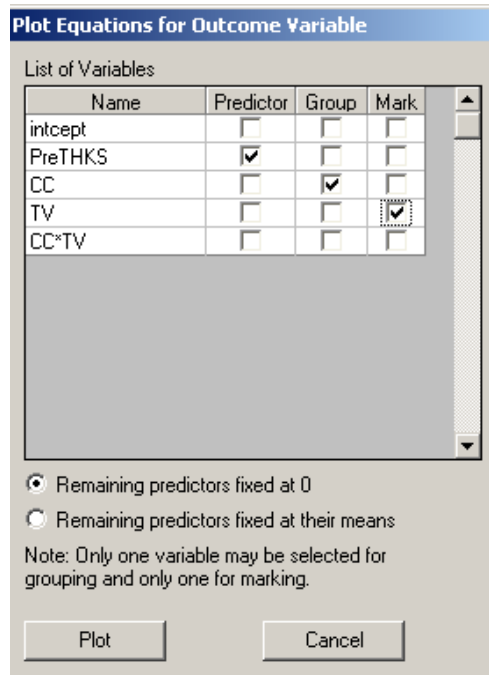


Figure XXX.3 shows slightly modified versions of the graphs obtained when the **Plot** button is clicked. For publication purposes, the line type associated with the value $TV = 1$ was changed to a dotted line. This was accomplished by clicking on the top line to activate the **Plot Parameters** dialog box and changing the line parameters so that the color is green and the style is dotted rather than solid. The plots show that the curriculum-based intervention had a larger effect on the post-intervention score: the intercept in the case where $CC = 1$ is appreciably higher than when $CC = 0$. In both graphs, the solid line indicates the absence of media-based intervention. The use of media-based intervention seems to have had a positive, albeit small, effect on the outcome.

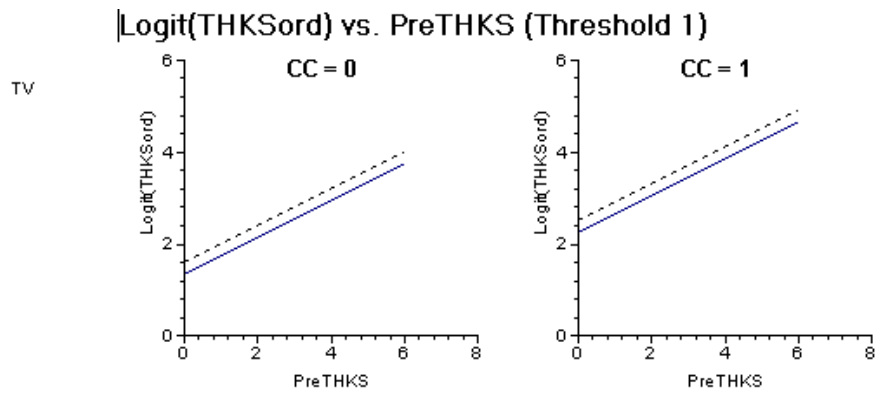


Figure XXX.3: Model-based graphs of THKSord by PreTHKS for groups

Interpretation of fixed effect estimates

The outcome variable has four categories, and there are thus 3 thresholds. The first of the thresholds, γ_1 , is set to zero in this analysis. The second and third thresholds are estimated under this model as $\hat{\gamma}_2 = 1.24187$ and $\hat{\gamma}_3 = 2.42044$. The coefficient β_1 , representing the effect of PreTHKS on THKSord, is estimated as 0.40331. The PreTHKS adjusted logit differences between CC = 1 and CC = 0 (keeping TV = 0) is estimated as $\hat{\beta}_2 = 0.92359$, in contrast with the PreTHKS adjusted logit differences between TV = yes and TV = no (keeping CC = 0) which is estimated as $\hat{\beta}_3 = 0.27485$. The coefficient β_4 denotes the adjusted difference in logit attributable to the interaction between CC and TV (CC*TV) and is estimated at -0.46570 , which diminishes the combined effects of CC and TV.

Logits for groups with no intervention

The first logit for groups with no intervention, for category 1 vs. categories 2 to 4, is $0 - \hat{\beta}_0 = -0.08855$. The second logit for the same group, for categories 1 and 2 vs. 3 and 4, can be calculated as $\hat{\gamma}_2 - \hat{\beta}_0 = 1.24187 - 0.08855 = 1.15332$. The third and final logit for this group, for categories 1 to 3 vs. 4, is $\hat{\gamma}_3 - \hat{\beta}_0 = 2.42044 - 0.08855 = 2.33189$. All of the logits are adjusted for the effect of the pre-intervention score PreTHKS.

Logits for groups with classroom curriculum intervention (CC = 1)

Turning to the groups with classroom curriculum intervention (CC = 1), logits can be obtained in similar fashion. $0 - \hat{\beta}_0 - \hat{\beta}_2 = -0.0885 - 0.92359 = -1.00909$, $\hat{\gamma}_2 - \hat{\beta}_0 - \hat{\beta}_2 = 1.15332 - 0.92359 = 0.22973$, and $\hat{\gamma}_3 - \hat{\beta}_0 - \hat{\beta}_2 = 1.4083$.

Logits for groups with media intervention (TV = 1)

For the groups where media intervention was employed, the logits are:

$$0 - \hat{\beta}_0 - \hat{\beta}_3 = -0.3634, \hat{\gamma}_2 - \hat{\beta}_0 - \hat{\beta}_3 = 0.87847, \text{ and } \hat{\gamma}_3 - \hat{\beta}_0 - \hat{\beta}_3 = 2.05704.$$

Estimated outcomes for groups: unit-specific results

To evaluate the expected effect of the CC, TV, CC*TV, and PreTHKS variables we use the expression below:

$$\log \left[\frac{\hat{P}(\text{THKSord}_{ij} \leq c)}{1 - \hat{P}(\text{THKSord}_{ij} \leq c)} \right] = \hat{\gamma}_c - \left[\hat{\beta}_0 + \hat{\beta}_2 \text{CC}_i + \hat{\beta}_3 \text{TV}_i + \hat{\beta}_4 (\text{CC}^* \text{TV})_i + \hat{\beta}_1 \text{PreTHKS}_{ij} \right]$$

$$= \hat{\gamma}_c - \left[0.08855 + 0.92359 \text{CC}_i + 0.27485 \text{TV}_i - 0.46570 (\text{CC}^* \text{TV})_i + 0.40331 (\text{PreTHKS}_{ij}) \right]$$

The variable PreTHKS is treated as a continuous variable in this example, although it too is originally a scale score. In order to facilitate comparison of treatment groups, the mean PreTHKS score for groups can be used to obtain the logits. The mean PreTHKS scores for each of the four treatment groups were given in Table XXX3. This table is reproduced below.

Table XXX3: Mean pre-intervention scores

Study condition	Proportion
CC = 0, TV = 0	2.152
CC = 0, TV = 1	2.087
CC = 1, TV = 0	2.050
CC = 1, TV = 1	1.979

The probabilities for the responses of typical subjects from the group with no intervention (TV = CC = 0) can be obtained using the modified equation

$$\log \left[\frac{\hat{P}(\text{THKSord}_{ij} \leq c)}{1 - \hat{P}(\text{THKSord}_{ij} \leq c)} \right] = \hat{\gamma}_c - [0.08855 + 0.40331(2.15202)] \\ = \hat{\gamma}_c - 0.95648.$$

Let

$$\hat{\eta}_{ijc} = \log \left[\frac{\hat{P}(\text{THKSor}_{ij} \leq c)}{1 - \hat{P}(\text{THKSor}_{ij} \leq c)} \right].$$

Similar equations for the groups with classroom curriculum intervention and media intervention respectively are then

$$\hat{\eta}_{ijc} = \hat{\gamma}_c - [0.08855 + 0.92359 + 0.40331(2.05000)] \\ = \hat{\gamma}_c - 1.83893$$

and

$$\hat{\eta}_{ijc} = \hat{\gamma}_c - [0.08855 + 0.27485 + 0.40331(2.086538)] \\ = \hat{\gamma}_c - 1.20492.$$

When both intervention methods were employed and thus TV = CC = CC*TV = 1, we have

$$\hat{\eta}_{ijc} = \hat{\gamma}_c - [0.08855 + 0.92359 + 0.27485 - 0.46570 + 0.40331(1.979112)] \\ = \hat{\gamma}_c - 1.619486.$$

In this example, the logistic link function was specified, and we can rewrite any formula of the form

$$\hat{\eta}_{ijc} = \gamma_c - a$$

in the alternative form

$$\hat{P}(\text{THKSord}_{ij} \leq c) = \frac{e^{\gamma_c - a}}{1 + e^{\gamma_c - a}} = \frac{e^{\eta_{ijc}}}{1 + e^{\eta_{ijc}}}.$$

Table XXX4 contains the cumulative response probabilities obtained through substitution in the above formulae for the first three categories of the ordinal outcome THKSord.

Table XXX4: Cumulative response probabilities for various groups and categories

Category	CC	TV	$\hat{\eta}_{ijc} = \gamma_c - \mathbf{x}'\boldsymbol{\beta}$	Probability of response
1	0	0	-0.9565	0.2776
1	1	0	-1.8389	0.1372
1	0	1	-1.2049	0.2306
1	1	1	-1.6195	0.1653
1 or 2	0	0	0.28539	0.5709
1 or 2	1	0	-0.5971	0.3550
1 or 2	0	1	0.0370	0.5092
1 or 2	1	1	-0.3776	0.4067
1, 2 or 3	0	0	1.4640	0.8121
1, 2 or 3	1	0	0.5815	0.6414
1, 2 or 3	0	1	1.2155	0.7713
1, 2 or 3	1	1	0.8010	0.6902

The probabilities reported in Table XXX4 are cumulative: for example, the probability of a response in either category 1 or 2 for the group with CC = TV = 0 is equal to 0.5709. The probability of a response in category 1 is 0.2776, and therefore the probability of a response in category 2 is $0.5709 - 0.2776 = 0.2933$. Similarly, the estimated response probability of a category 3 response for a respondent from the same group is $0.8121 - 0.5709 = 0.2412$. To obtain the category 4 response for a respondent from the first group, the value of the estimated response in categories 1, 2, or 3 has to be subtracted from 1, so that the probability of a response in category

4 for a typical respondent with $CC = TV = 0$ is $1 - 0.8121 = 0.1879$. The cumulative probabilities for the first 3 categories of the ordinal outcome are plotted in Figure XXX.4.

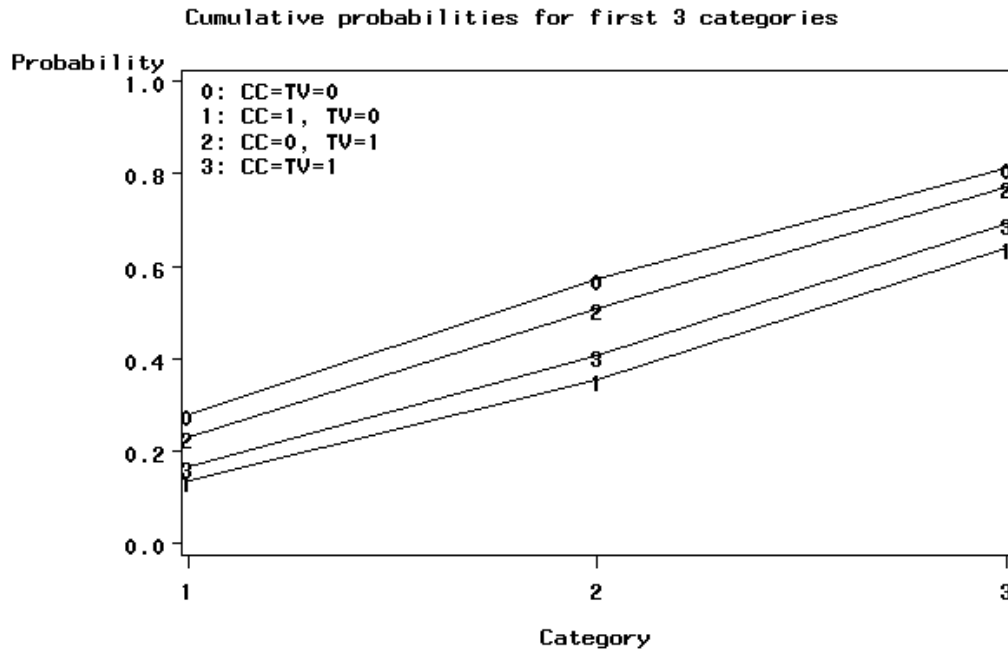


Figure XXX.4: Cumulative response probabilities for categories 1 to 3 of THKSord

The graph shows two groupings: one representing $CC = 0$, regardless of the value of TV ; and the other $CC = 1$, again regardless of the value of TV . The smallest probability to fall in categories other than category 1 (normal) is for the combination $CC = TV = 1$. The fact that the plotted cumulative probability lines for $CC = 1$ and $TV = 1$ are close to the line for $CC = TV = 1$ suggests that the implementation of media intervention ($TV = 1$ if implemented) has less impact on the outcome than the use of a classroom curriculum ($CC = 1$ if implemented).

To obtain category probabilities, differences between the cumulative probabilities obtained above are calculated. In other words,

$$\hat{P}(\text{THKSord}_{ij} = c) = \hat{P}(\text{THKSord}_{ij} \leq c) - \hat{P}(\text{THKSord}_{ij} \leq c - 1)$$

The category probabilities are reported in Table XXX5 and are graphically displayed in Figure XXX.5.

Table XXX5: Estimated unit-specific probabilities for THKSord categories

Category	CC	TV	$\hat{P}(\text{THKSord}_{ij} = c)$
1	0	0	0.2776
	1	0	0.1372
	0	1	0.2306
	1	1	0.1653
2	0	0	0.2933
	1	0	0.2178
	0	1	0.2786
	1	1	0.2414
3	0	0	0.2413
	1	0	0.2864
	0	1	0.2620
	1	1	0.2835
4	0	0	0.1879
	1	0	0.3586
	0	1	0.2287
	1	1	0.3098

A typical respondent from the control group (no intervention) was less likely to respond in categories 3 or 4 of the ordinal post-intervention outcome variable. For both this group and the group which was assigned to media intervention only, the most likely response was in category 2 and the least likely response in category 4. In contrast, groups that were subjected to the classroom curriculum intervention, with or without media intervention, were most likely to display a high level of knowledge (i.e., a response in categories 3 or 4), and least likely to respond in the first category. From this graph we conclude that the classroom curriculum intervention was key – groups subjected to the intervention tended to increase in knowledge over the study period.

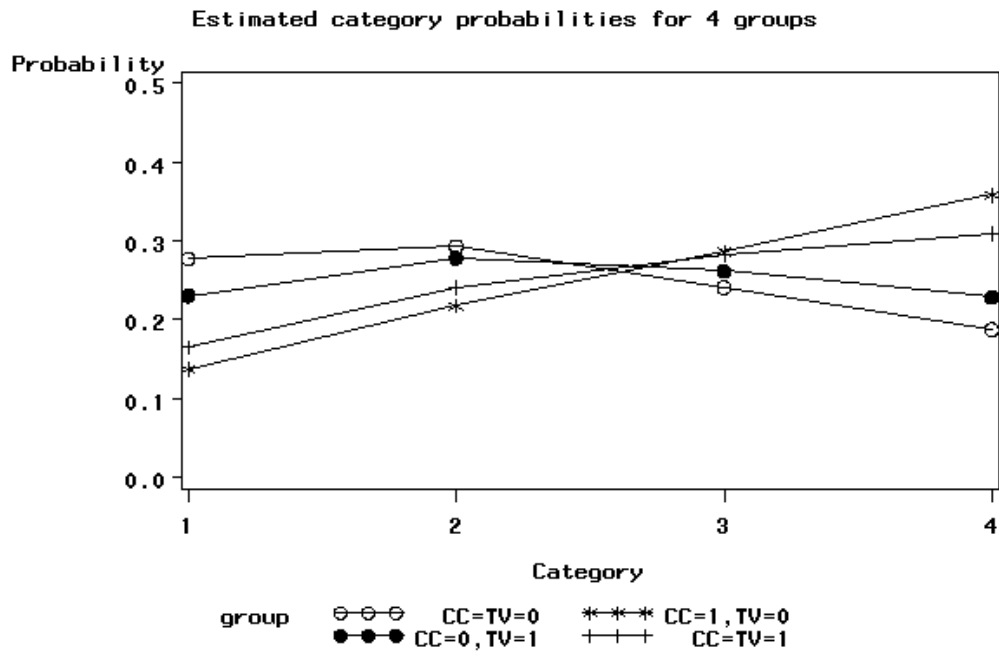


Figure XXX.5: Estimated category probabilities for THKSord

Estimated outcomes for different groups: population-average results

In the introduction to this section, we defined the latent response variable model as

$$y_{ij} = \mathbf{z}'_{(1)ij} \mathbf{b}_i + \mathbf{x}'_{(1)ij} \boldsymbol{\beta}_{(1)} + \varepsilon_{ij},$$

making the assumption that $\varepsilon_{ij} \sim i.i.d.(0, \sigma^2)$. For a probit link function $\sigma_\varepsilon^2 = 1$, and for a logistic link function it is assumed that $\sigma_\varepsilon^2 = \pi^2/3$, as indicated in the final lines of the output file. Under the assumption that \mathbf{v}_i and ε_{ij} are independently distributed, it follows that

$$\sigma_{y_{ij}}^2 = \mathbf{z}'_{ij} \Phi_{v_i} \mathbf{z}_{ij} + \sigma_\varepsilon^2.$$

Let

$$d_{ij} = \frac{\sigma_{y_{ij}}^2}{\sigma_{\varepsilon}^2}.$$

The quantity d_{ij} is called the design effect. The estimated population-average probabilities (Hedeker & Gibbons, 2006) are obtained in a similar fashion as the unit-specific probabilities, but replacing $\hat{\eta}_{ijc}$ with $\hat{\eta}_{ijc}^* = \hat{\eta}_{ijc} / \sqrt{d_{ij}}$.

From the output, we have $\text{var}(u_{i0}) = (0.27118)^2 = 0.074$, where u_{i0} denotes the random intercept coefficient. In this case, $\mathbf{z}'_{ij} = \mathbf{1}$ and hence, with $\sigma_{\varepsilon}^2 = \pi^2 / 3$ for the logistic link,

$$\sigma_{y_{ij}}^2 = 1 \times 0.074 + (3.1416)^2 / 3 = 3.3639.$$

Therefore

$$d_{ij} = \frac{3.3639}{3.2899} = 1.0225.$$

To obtain the population-average probability estimates, we now replace the η_{ijc} values calculated for the unit-specific case with $\hat{\eta}_{ijc}^* = \hat{\eta}_{ijc} / \sqrt{d_{ij}}$.

We can compare these estimated population-average probabilities with the observed data for the four groups formed by the categories of TV and CC as shown in Table XXX4 previously. Table XXX6 shows the estimated population-average probabilities.

A comparison of these probabilities with those reported in Table XXX4 shows little difference between the unit-specific and population-average category probabilities for treatment groups. The population-average category probabilities for the first two categories are slightly smaller than the corresponding unit-specific probabilities, while those for category 3 are slightly larger. The extent of differences between unit-specific and population-average results are highly dependent on the "scaling"

induced by dividing the η_{ijc} s by $\sqrt{d_{ij}}$. In the current example, $\sqrt{d_{ij}} = 1.0112$ and thus no large differences could be expected. To obtain category probabilities, differences between the cumulative probabilities may be calculated, as illustrated in the case of the unit-specific results.

Table XXX6: Cumulative response probabilities for various groups and categories

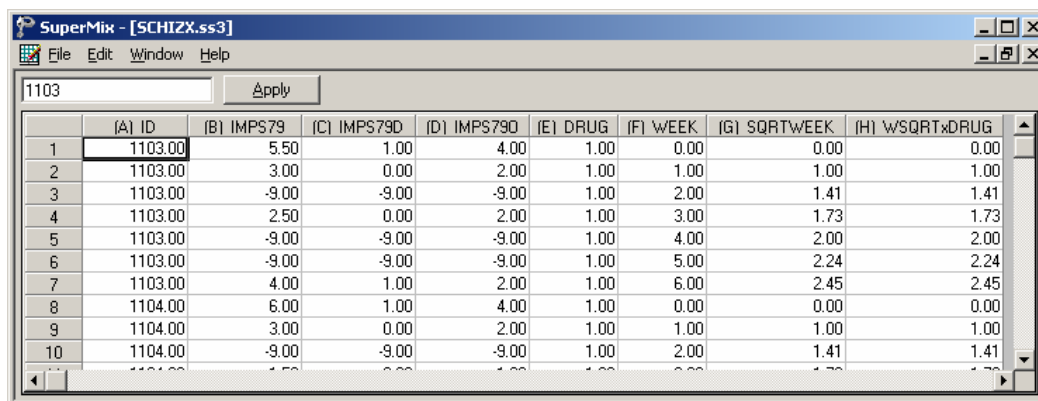
Category	CC	TV	$\hat{\eta}_{ijc}^* = \hat{\eta}_{ijc} / \sqrt{d_{ij}}$	Probability of response
1	0	0	-0.9565/1.0112	0.2797
1	1	0	-1.8389/1.0112	0.1396
1	0	1	-1.2049/1.0112	0.2330
1	1	1	-1.6195/1.0112	0.1678
1 or 2	0	0	0.28539/1.0112	0.5701
1 or 2	1	0	-0.5971/1.0112	0.3565
1 or 2	0	1	0.0370/1.0112	0.5092
1 or 2	1	1	-0.3776/1.0112	0.4077
1, 2 or 3	0	0	1.4640/1.0112	0.8097
1, 2 or 3	1	0	0.5815/1.0112	0.6399
1, 2 or 3	0	1	1.2155/1.0112	0.7689
1, 2 or 3	1	1	0.8010/1.0112	0.6883

1.2 Two-level ordinal analysis of SCHIZX data

1.2.1 The data

To illustrate the application of the mixed-effects ordinal logistic regression model to longitudinal data, we examined data collected in the NIMH Schizophrenia Collaborative Study on treatment-related changes in overall severity. Specifically, Item 79 of the Inpatient Multidimensional Psychiatric Scale (IMPS; Lorr & Klett, 1966) was used. In this study, patients were randomly assigned to receive one of four medications: placebo, chlorpromazine, fluphenazine, or thioridazine. Since previous analyses (Longford, 1993, and Gibbons & Hedeker, 1994) revealed similar effects for the three anti-psychotic drug groups, they were combined in the present analysis. Finally, again based on previous analysis, a square root transformation of time was chosen to linearize the relationship of the IMPS79 scores over time.

Data for the first 10 observations are shown below in the form of a SuperMix spreadsheet file, named **schizx.ss3**.



	(A) ID	(B) IMPS79	(C) IMPS79D	(D) IMPS79O	(E) DRUG	(F) WEEK	(G) SQRTWEEK	(H) WSQRTxDRUG
1	1103.00	5.50	1.00	4.00	1.00	0.00	0.00	0.00
2	1103.00	3.00	0.00	2.00	1.00	1.00	1.00	1.00
3	1103.00	-9.00	-9.00	-9.00	1.00	2.00	1.41	1.41
4	1103.00	2.50	0.00	2.00	1.00	3.00	1.73	1.73
5	1103.00	-9.00	-9.00	-9.00	1.00	4.00	2.00	2.00
6	1103.00	-9.00	-9.00	-9.00	1.00	5.00	2.24	2.24
7	1103.00	4.00	1.00	2.00	1.00	6.00	2.45	2.45
8	1104.00	6.00	1.00	4.00	1.00	0.00	0.00	0.00
9	1104.00	3.00	0.00	2.00	1.00	1.00	1.00	1.00
10	1104.00	-9.00	-9.00	-9.00	1.00	2.00	1.41	1.41

The variables of interest are:

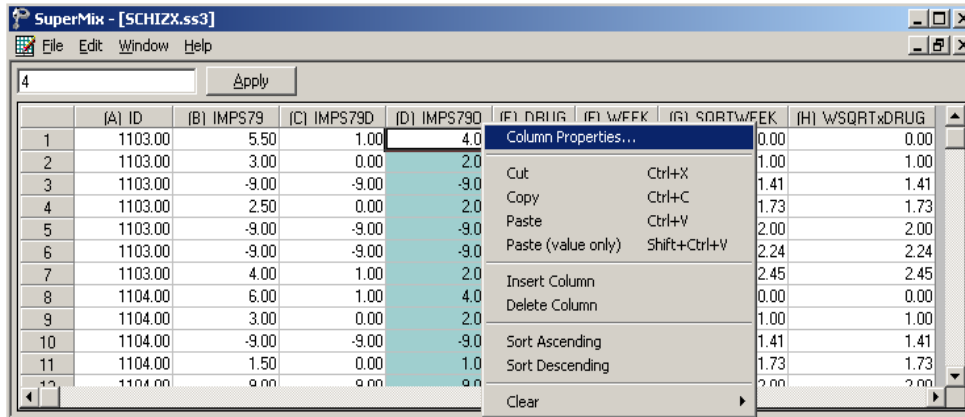
- ID indicates the subject (437 patients in total).
- IMPS79 represents the original score on Item 79 of the Inpatient Multidimensional Psychiatric Scale. It was scored as: 1 = normal, or not at all ill; 2 = borderline mentally ill; 3 = mildly ill; 4 = moderately ill; 5 = markedly ill; 6 = severely ill; and 7 = among the most extremely ill.

- IMPS79D is a recoded version of the same scale, but in binary form, where scores up to, but excluding 3.5 were coded 0, and scores of 3.5 or higher were coded 1. The value "0" is associated with measurements classified as normal, borderline, mildly, or moderately mentally ill, while the value "1" was assigned to measurements corresponding to "markedly ill" through "most extremely ill."
- IMPS79O is also a recoded version of the same scale, but with the 7 original categories reduced to four: 1 = normal or borderline mentally ill, 2 = mildly or moderately ill, 3 = markedly ill, and 4 = severely or among the most extremely ill.
- DRUG indicates the treatment group, where 0 indicates the placebo patients, and 1 refers to the drug patients.
- WEEK represents the time during the course of the study when a specific measurement was made, and ranges between 0 and 6.
- SQRTWEEK is the square root of WEEK. This variable is generated within the SuperMix spread sheet. For more information, please refer to section XXXX.
- WSQRTxDRUG is the product of the treatment group and the square root of WEEK.

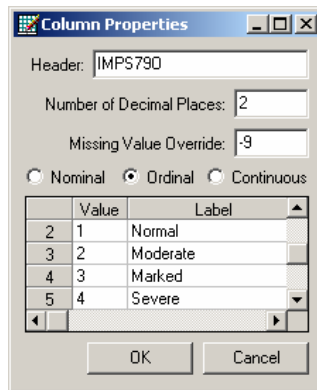
In this data file, each subject's data consist of seven lines, these being the repeated measurements on seven occasions. Notice that there are missing value codes (-9) for some subjects at specific time points. The data from these time points will not be used in the analysis, but data from these subjects at other time points where there are no missing data will be used in the analysis. Thus, for inclusion into the analysis, a subject's data (both the dependent variable and all model covariates being used in a particular analysis) at a specific time point must be complete. The number of repeated observations per subject then depends on the number of time points for which there are non-missing data for that subject. The specification of missing data codes will be illustrated in the model specification section to follow.

1.2.1.1 Defining column properties

Defining column properties for the ordinal data is recommended. We use the column of IMPS79O as an example. First, highlight the column of IMPS79O by clicking on its header. Then right click and select the **Column Properties** option as shown below to open the **Column Properties** dialog box.



The header of the **Column Properties** dialog box indicates the current variable name. Keep the default number of decimal places unchanged. Enter -9 in the **Missing Value Override** string box. Select the **Ordinal** radio button to activate the grid field to enter the labels for each category as shown below.



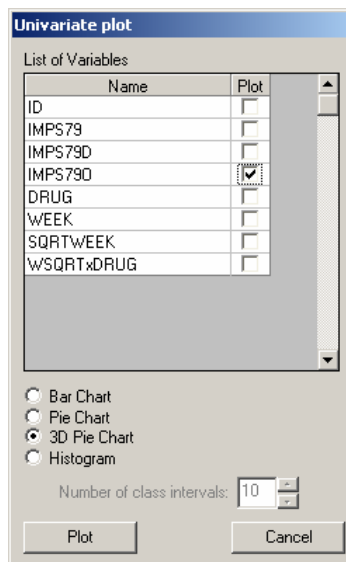
Click on the **OK** button and save the change to the data set by clicking on the **File, Save** option.

The outcome variable: univariate graphs

As a first step, we take a look at the ordinal variable IMPS790 which is the potential dependent variable in this study.

Pie chart

To generate a pie chart for IMPS790, first open the **schizx.ss3** in the SuperMix spreadsheet. Next, select the **File, Data-based Graphs, Univariate** option to load the **Univariate plot** dialog box. Select the variable IMPS790 and indicate that a **3D Pie Chart** is to be graphed as shown below.



Click the **Plot** button to display the following pie chart. Note that most of the observations fall into the Severe illness category. Keep in mind that the pie chart takes all observations, regardless of the time of measurement, into account. As such, it is informative about the distribution of all observed values of the potential outcome, but does not provide any information on possible trends in illness level over time.

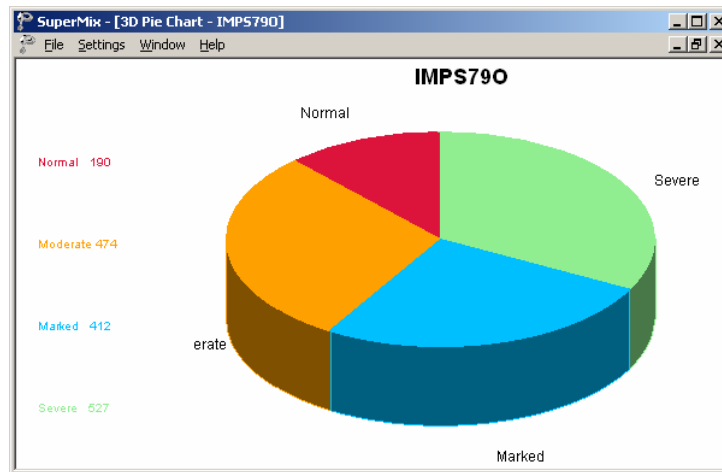


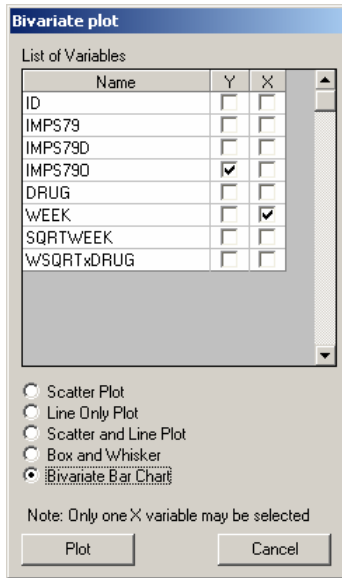
Figure XXX.6: Pie chart of IMPS790 values

Relationships between variables: bivariate bar chart

It is hoped that the severity of the illness (IMPS790) will decrease over the treatment period. Before considering fitting a model to these data, we would like to explore the relationship between IMPS790 and WEEK using a bivariate bar chart.

Bivariate bar chart

A bivariate bar chart is accessed via the **Data-based Graphs, Bivariate** option on the **File** menu. The **Bivariate plot** dialog box is completed as below: select the outcome variable IMPS790 as the **Y**-variable of interest, and the predictor WEEK to be plotted on the **X**-axis. Check the **Bivariate Bar Chart** option, and click **Plot**.



As shown below, most patients did not participate in the study at weeks 2, 4 and 5. At the beginning of the study (week 0), a large percentage of patients are markedly or severely ill. By the end of the study (week 6), most patients are reported as normal or moderate.

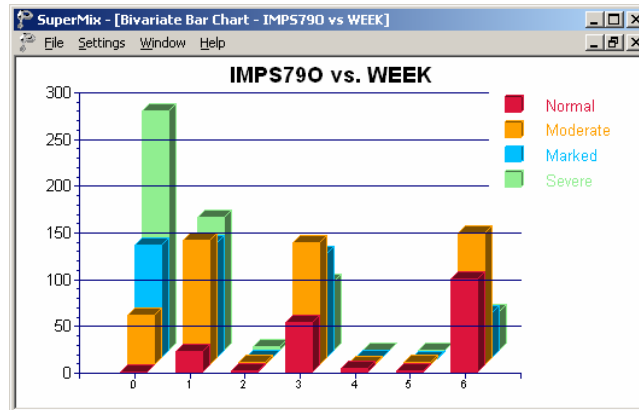


Figure XXX.7: Bar chart of IMPS790 vs. WEEK

1.2.2 An ordinal regression model with random intercept

1.2.2.1 Introduction

As discussed in Section XX.1.1, an ordinal variable is a categorical variable where there is a logical ordering to the categories. In most cases, treating an ordinal outcome as a continuous variable is inadvisable, due to the reasons discussed in Section XX.1.1. As in the case of a binary outcome variable, a link function is used in order to take the ceiling and floor effects of the ordinal outcome into account. The available link functions in SuperMix include probit, logistic, complementary log-log and log-log. Detailed information on these link functions are given in section XXXX.

1.2.2.2 The model

Let the outcome variable be coded into c categories, where $c = 1, 2, \dots, C$. In this example, the ordinal variable IMPS790 defines the severity of the illness in terms of four categories, and thus $C = 4$. As ordinal models utilize cumulative comparisons of the categories, define the cumulative probabilities for the C categories of the outcome Y as $P_{ijc} = \Pr(Y_{ij} \leq c) = \sum_{k=1}^c p_{ijk}$, where p_{ijk} represents the probability that the response of the j th measurement on patient i occurs in category k .

The type of drug, time elapsed since start of treatment, and the interaction between drug taken and time elapsed are of interest as predictors. The logistic regression model with IMPS790 as outcome can then be written as

Level 1 model:

$$y_{ij} = \log\left(\frac{P_{ijc}}{1 - P_{ijc}}\right) = \gamma_c - \left[b_{0i} + b_{1i}\text{DRUG}_i + b_{2i}\text{SQRTWEEK}_i + b_{3i}(\text{WSQRT} \times \text{DRUG})_i \right],$$
$$j = 1, \dots, n_i; c = 1, 2, \dots, C - 1$$

Level 2 model:

$$\begin{aligned} b_{0i} &= \beta_0 + v_{0i}, \quad i = 1, \dots, N \\ b_{1i} &= \beta_1 \\ b_{2i} &= \beta_2 \\ b_{3i} &= \beta_3 \end{aligned}$$

The cumulative probability can be expressed by

$$P_{ijc} = \frac{e^{\gamma_c - [b_{0i} + b_{1i} \text{DRUG}_i + b_{2i} \text{SQRTWEEK}_i + b_{3i} (\text{WSQRT} \times \text{DRUG})_i]}}{1 + e^{\gamma_c - [b_{0i} + b_{1i} \text{DRUG}_i + b_{2i} \text{SQRTWEEK}_i + b_{3i} (\text{WSQRT} \times \text{DRUG})_i]}}$$

To obtain the probability for category c ,

$$p_{ij,c} = P_{ij,c+1} - P_{ij,c}$$

As shown above, the intercept b_{0i} is estimated by a level-2 equation. It indicates that patient i 's initial IMPS790 value is not only determined by the population average β_0 , but also by the patient difference v_{0i} . In other words, patients may have different average intercepts, and the model makes provision for this eventuality. The slopes are assumed to be the same for all the patients, which implies that each patient's trend line is parallel to the population trend.

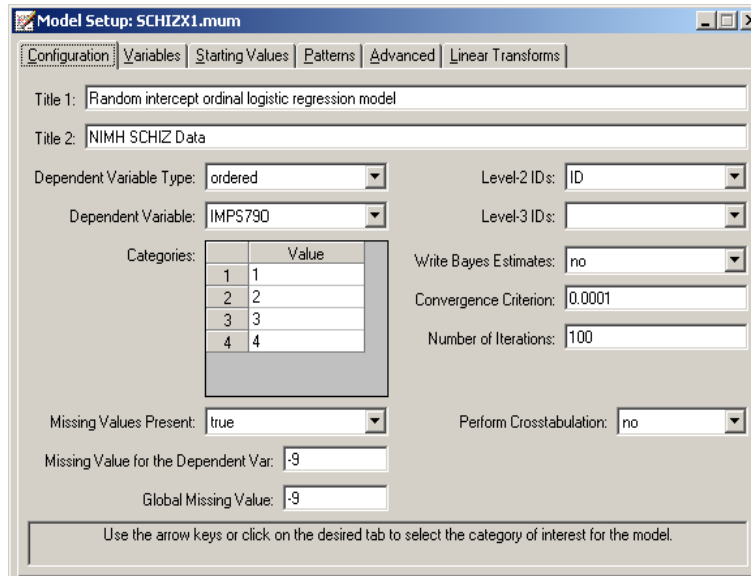
The connection between an ordinal outcome variable y with C categories and an underlying continuous variable y^* is

$$y = c \leftrightarrow \gamma_{j-1} \leq y^* \leq \gamma_j, \quad c = 1, 2, \dots, C$$

where it is assumed that $\gamma_0 = -\infty$ and $\gamma_C = +\infty$. In addition, γ_1 is usually set to 0 to avoid identification problems.

1.2.2.3 Setting up the analysis

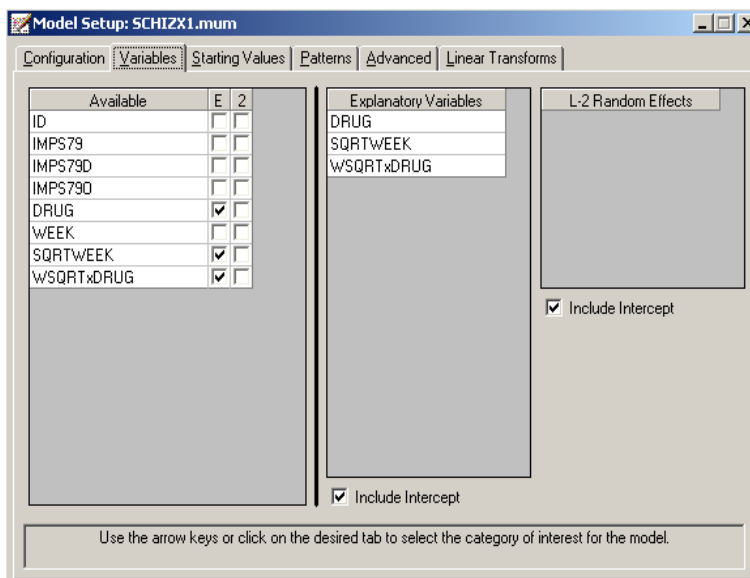
Open the SuperMix spreadsheet **schizx.ss3** and select the **File, New Model Setup** option. In the **Configuration** screen of the **Model Setup** window, enter a title for the analysis in the **Title** text boxes. Select **ordered** from the **Dependent Variable Type** drop-down list box. Select the outcome variable **IMPS790** from the **Dependent Variable** drop-down list box. Once this selection has been made, the **Categories** grid is displayed, with the distinct values of the categories shown.



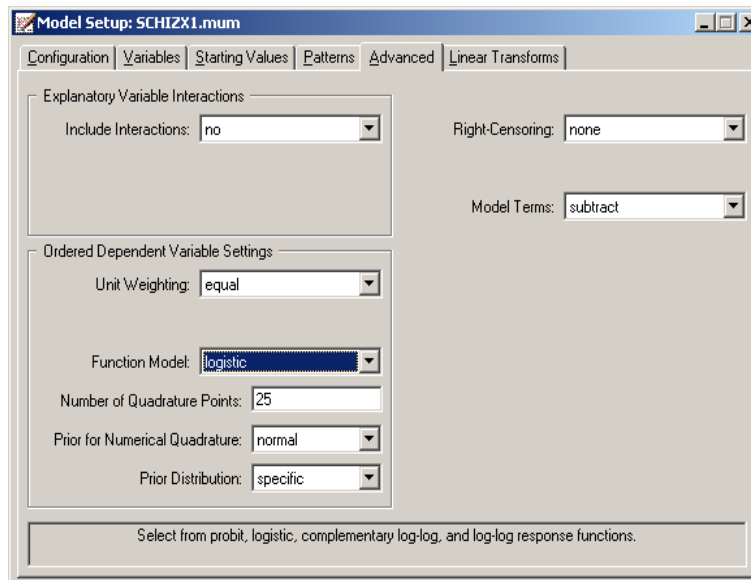
We notice that the missing value -9 is also included as a category. The **Missing Values Present** drop-down list box is used to specify the values of missing data for both outcome and predictors. As a first step, set the value of the **Missing Values Present** drop-down list box to **True**. The appearance of the screen will change when this is done, and text boxes for the specification of the missing data codes are displayed. Start by entering the value -9 in the **Missing Value for the Dependent Var** text box. Do the same for all the predictors included in the model by entering -9 in the **Global Missing Value** text box. Finally, select the patient ID from **Level-2 IDs** drop-down list box to produce the **Configuration** screen seen above.

Proceed to the **Variables** screen by clicking on this tab. The **Variables** tab is used to specify the fixed and random effects to be included in the model. Select **DRUG**, **SQRTWEEK** and **WSQRTxDRUG** as explanatory (fixed) variables using the **E** check

boxes next to the variables names in the **Available** grid at the left of the screen. The **Include Intercept** check box in the **Explanatory Variables** grid is checked by default, indicating that an intercept term will automatically be included in the fixed part of the model. Next, specify the random effects at level 2 of the hierarchy. In this example, we want to fit a model with random intercepts at level 2. By default, the **Include Intercept** check box in the **L-2 Random Effects** is checked, indicating the inclusion of a random intercept at this level in the model.



The default link function for the ordinal outcome variable is the probit link function. To change it to the logistic link function corresponding to the model formulation above, click on the **Advanced** tab and select the **logistic** link function from the **Function Model** drop-down list box as shown below. Use 25 quadrature points.



Before running the analysis, the model specifications have to be saved. Select the **File, Save As** option, and provide a name (**SCHIZX1.mum**) for the model specification file. Run the analysis by selecting the **Run** option from the **Analysis** menu.

1.2.2.4 Discussion of results

Syntax

The syntax corresponding to the model setup is given in the **model specifications**. These lines of SuperMix syntax are saved as a ***.inp** file with the same name as the model setup file (***.mum**). At the top of the output file, the syntax lines are printed as shown below.

The first part indicates that an ordinal outcome is analyzed, states the selection of iteration control options, does not request Bayes residuals, and contains all the specifications necessary to define the model fitted as an ordinal model with logistic link function. The second part of the syntax provides information on the structure of the data, the name and structure of the outcome variable, the missing values and the predictors included in the model.

```

SuperMix - [SCHIZX1.out]
File Analysis Window Help

The following lines were read from file SCHIZX1.inp

Model=Ordinal;
Options Converge=0.0001 Maxiter=100 Bayes=No ModelTerms=subtract
      NQuadPTS=25 Prior=normal;
Link=logistic;
Varnames= ID IMPS79 IMPS79D IMPS790 DRUG WEEK SQRTWEEK WSQRTxDR intcept;
Title=Random intercept ordinal logistic regression model;
Title2=NIMH SCHIZ data;
DataFile=C:\Program Files\SuperMix\SCHIZX1.dat;
Level2ID= ID;
Dependent= IMPS790;
Categories= 1 2 3 4;
Dependent_Miss=-9;
Global_Miss=-9;
Predictors= intcept DRUG SQRTWEEK WSQRTxDR;
L2Random= intcept;
FixPatType=Free;
Cov2PatType=Correlated;

Save As... Close

```

Data summary

The next section of the output file contains a description of the hierarchical structure and model specifications. The use of a logistic response function (logit link function) with the assumption of a normal distribution of random effects is indicated. This is followed by a summary of the number of observations nested within each patient. As shown below, 437 patients with a total of 1603 observations are included in this study after listwise deletion. The number of observations per patient (level 2 unit) varies between 2 and 5.

```

SuperMix - [SCHIZX1.out]
File Analysis Window Help

Level 1 observations = 1603
Level 2 observations = 437

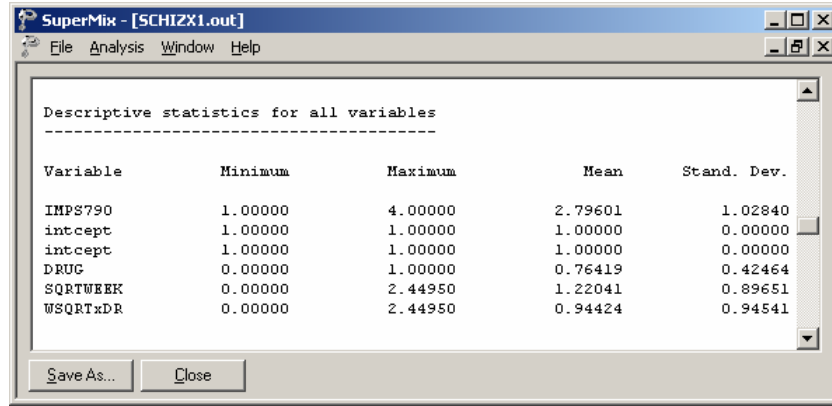
The number of level 1 observations per level 2 unit are:

  4  4  3  4  4  4  4  4  4  3  4  4  4  2  3  4  3  4  3
  4  4  4  3  3  2  4  4  4  4  4  3  4  4  4  4  4  4  4
  4  4  2  3  4  3  4  4  4  3  4  4  2  2  4  5  4  2  4
  4  3  4  4  3  2  3  4  4  4  4  4  2  4  4  4  5  4
  4  2  2  4  2  4  4  3  3  4  4  4  4  4  4  4  4  3  3
  4  2  3  4  4  4  2  5  3  4  4  2  4  4  4  2  4  4  4
  4  4  4  4  4  4  5  2  4  3  4  4  2  2  4  4  4  4  4
  2  4  4  4  4  4  4  4  4  4  4  4  4  2  4  4  2  4  4
  4  4  3  4  2  4  4  3  2  3  4  4  3  3  4  3  4  4  4
  4  4  4  4  4  4  4  4  4  4  4  2  3  3  5  4  3  4  4
  3  ?  4  4  4  4  4  3  3  4  4  4  4  4  4  4  4  4  4

```

Descriptive statistics and starting values

Next, the descriptive statistics for all the variables are given. Notice that the variable name WSQRTxDRUG is truncated to WSQRTxDR. This is because SuperMix only recognizes the first 8 characters of a variable name.



SuperMix - [SCHIZX1.out]

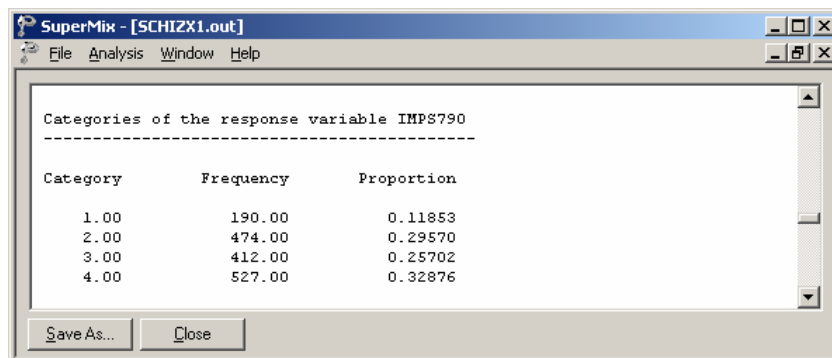
File Analysis Window Help

Descriptive statistics for all variables

Variable	Minimum	Maximum	Mean	Stand. Dev.
IMPS790	1.00000	4.00000	2.79601	1.02840
intcept	1.00000	1.00000	1.00000	0.00000
intcept	1.00000	1.00000	1.00000	0.00000
DRUG	0.00000	1.00000	0.76419	0.42464
SQRTWEEK	0.00000	2.44950	1.22041	0.89651
WSQRTxDR	0.00000	2.44950	0.94424	0.94541

Save As... Close

As shown below, the output file for the ordinal outcome also provides a frequency table for the dependent variable. The data summary is followed by descriptive statistics for all the variables included in the model (not shown). We note that 33% of the measurements were in the highest category of the outcome variable, and correspond to the "severely or among the most extremely ill" group. Only 12% of measurements are in the first category ("normal, not at all ill").



SuperMix - [SCHIZX1.out]

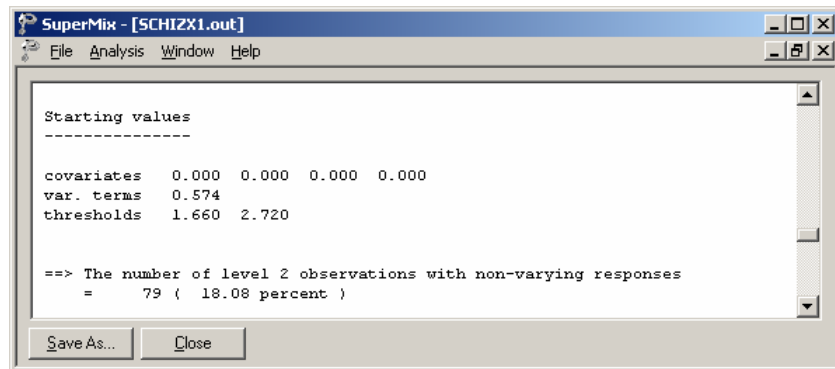
File Analysis Window Help

Categories of the response variable IMPS790

Category	Frequency	Proportion
1.00	190.00	0.11853
2.00	474.00	0.29570
3.00	412.00	0.25702
4.00	527.00	0.32876

Save As... Close

Descriptive statistics are followed by the starting values of parameters. The starting values for the predictors intercept, DRUG, SQRTWEEK and WSQRTxDR are given in the first line (covariates), while the starting value for the variance component associated with the random level-2 intercept is given in the second line (var. terms). The third line shows the starting values of the thresholds. In 18% of the subjects, no change in the category assigned for measurements was observed, as indicated by the last two lines shown below. Since the first threshold is fixed at 0 for identification purposes, starting values for the second and third thresholds only are listed.



```
SuperMix - [SCHIZX1.out]
File Analysis Window Help

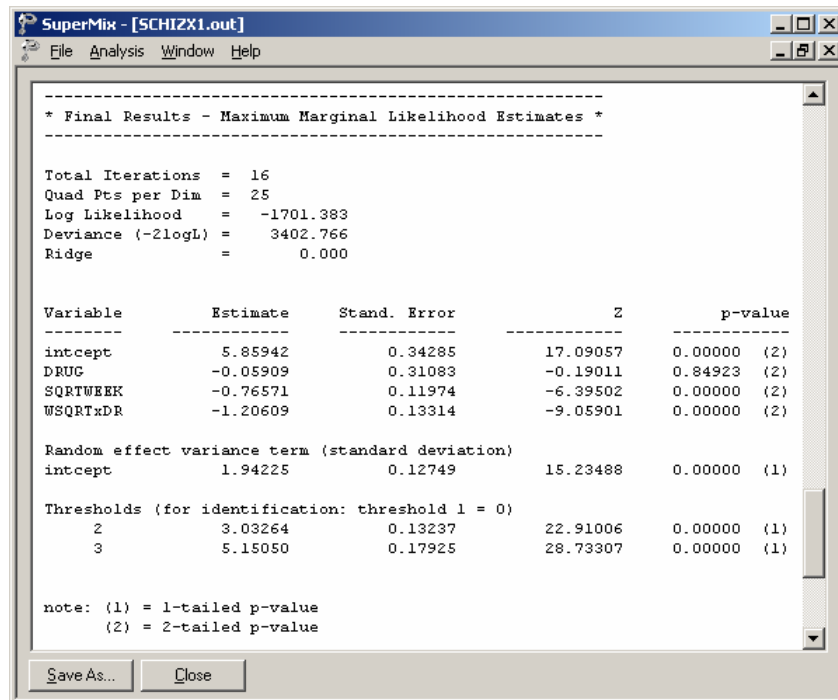
Starting values
-----
covariates  0.000  0.000  0.000  0.000
var. terms  0.574
thresholds  1.660  2.720

==> The number of level 2 observations with non-varying responses
    =    79 ( 18.08 percent )

Save As... Close
```

Fixed effects estimates

The final results after 16 iterations are shown next. The estimates are shown in the column with heading Estimate, and correspond to the coefficients $\beta_0, \beta_1, \dots, \beta_3$ in the model specification. The standard error, Z-value and p -value are also printed.



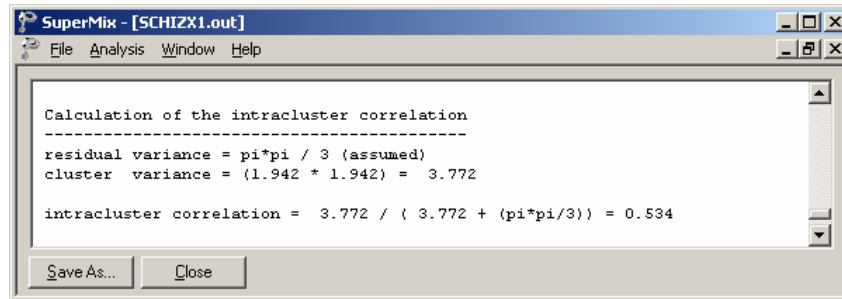
The variation in the intercept over the subjects is estimated as $1.94225^2 = 3.77233$, and from the associated p -value we conclude that there is significant variation in the (random) intercept between the patients included in this analysis. In the case of the fixed effects, a 2-tailed p -value is used, as the alternative hypothesis considered here is of the form $H_1: \beta \neq 0$. As variances are constrained to be elements of the interval $[0, +\infty)$ and thresholds are constrained so that $\gamma_1 \leq \gamma_2 \leq \gamma_3$, the p -values used for these effects are 1-tailed. The results indicate that the treatment groups do not differ significantly at baseline (the estimated DRUG coefficient is not significant). The placebo group seems to improve over time, as the SQRTWEEK coefficient is both significant and negative. Note that the interpretation of the main effects depends on the coding of the variable, and on the significance of the WSQRTxDR interaction which forms part of the model.

As noted before, it is assumed that $\gamma_0 = -\infty$ and $\gamma_C = +\infty$, with γ_1 usually set to 0 to avoid identification problems. For the present example, $C = 4$, and from the output we see that $\hat{\gamma}_2 = 3.03264$ and $\hat{\gamma}_3 = 5.15150$. These values are used in combination with the coefficients of DRUG, SQRTWEEK, and WSQRTxDR to

calculate estimated outcomes for different groups of patients (see Section XXX).

Intraclass correlation (ICC)

Below the estimate the intraclass correlation (ICC) is given. The residual variance for the logistic link function is assumed to be $\pi^2/3$.



The ICC in this model refers to the intra-person correlation. It is reported as 0.534, which is fairly high. Generally, the shorter the interval between the repeated measurements, the higher the ICCs will be.

1.2.2.5 Interpreting the output

Estimated outcomes for groups: unit-specific probabilities

To evaluate the expected effect of the treatment group and the square root of time of treatment, while allowing for the interaction between treatment and the square of time, we use the expression below:

$$\log\left(\frac{\hat{P}_{ijc}}{1-\hat{P}_{ijc}}\right) = \hat{\gamma}_c - \left[\hat{b}_{0i} + \hat{b}_{1i}\text{DRUG}_i + \hat{b}_{2i}\text{SQRTWEEK}_i + \hat{b}_{3i}(\text{WSQRT}\times\text{DRUG})_i \right]$$

or, in the notation introduced in Section XXX,

$$\begin{aligned} \log\left(\frac{\hat{P}_{ijc}}{1-\hat{P}_{ijc}}\right) &= \hat{\eta}_{ijc} \\ &= \hat{\gamma}_c - 5.85942 + 0.05909 \times \text{DRUG}_i + 0.76571 \times \text{SQRTWEEK}_i \\ &\quad + 1.20609 \times (\text{WSQRT} \times \text{DRUG})_i. \end{aligned}$$

When $c = 1$ and $\gamma_1 = 0$, we find that, for a patient from the control group ($\text{DRUG} = 0$, $\text{SQRTWEEK} = \text{WSQRT} \times \text{DR} = 0$),

$$\begin{aligned} \log\left(\frac{\hat{P}_{ij1}}{1-\hat{P}_{ij1}}\right) &= \hat{\eta}_{ij1} = 0 - 5.85942 \\ \hat{P}_{ij1} &= \frac{e^{\hat{\eta}_{ij1}}}{1 + e^{\hat{\eta}_{ij1}}} = 0.002844 \end{aligned}$$

Similarly, the probabilities that a typical patient from the control group responded in a specific category at the start of the study are obtained by substituting $\gamma_1 = 0$ with $\hat{\gamma}_2 = 3.03264$, and $\hat{\gamma}_3 = 5.15050$. The cumulative probabilities we calculated are

$$\begin{aligned} \hat{P}_{ij2} &= \frac{e^{\hat{\eta}_{ij2}}}{1 + e^{\hat{\eta}_{ij2}}} = \frac{e^{3.03264 - 5.85942}}{1 + e^{3.03264 - 5.85942}} = 0.05589 \\ \hat{P}_{ij3} &= \frac{e^{\hat{\eta}_{ij3}}}{1 + e^{\hat{\eta}_{ij3}}} = \frac{e^{5.15050 - 5.85942}}{1 + e^{5.15050 - 5.85942}} = 0.32984 \end{aligned}$$

Thus, the estimated category probabilities we have for such a group (category 1 to 4) are obtained as

$$\hat{p}_{ij1} = 0.00284 - 0 = 0.00284$$

$$\hat{p}_{ij2} = 0.05589 - 0.00284 = 0.05305$$

$$\hat{p}_{ij3} = 0.32984 - 0.05589 = 0.27394$$

$$\hat{p}_{ij4} = 1 - 0.32984 = 0.67016$$

For this group of patients (DRUG = 0) at the starting week, the expected percentages of patients in each of the categories are as follows: 0.3% of the patients are normal or borderline mentally ill; 5.3% of the patients are mildly or moderately ill; 27.4% are markedly ill and 67% are severely or extremely ill. Similarly, we can calculate the estimated percentages for both groups at all the time points as shown in Table XXX7.

Table XXX7: Estimated % for both groups at 7 time points

severity	Placebo patients (drug = 0)				Drug patients (drug = 1)			
	normal	moderate	marked	severe	normal	moderate	marked	severe
week 0	0.28%	5.30%	27.39%	67.02%	0.30%	5.61%	28.39%	65.70%
week 1	0.61%	10.68%	40.13%	48.58%	2.13%	28.96%	47.86%	21.05%
week 2	0.84%	14.05%	44.36%	40.76%	4.69%	45.83%	38.94%	10.54%
week 3	1.06%	17.17%	46.73%	35.04%	8.43%	57.21%	28.43%	5.92%
week 4	1.30%	20.19%	47.98%	30.52%	13.51%	62.91%	20.00%	3.58%
week 5	1.56%	23.15%	48.47%	26.83%	19.92%	63.85%	13.95%	2.28%
week 6	1.83%	26.04%	48.39%	23.75%	27.48%	61.24%	9.78%	1.51%

The above table can be graphically represented in the following bar charts. It clearly shows that the numbers of markedly and severely ill patients decrease dramatically over time. The improvement for the drug patients is larger than the placebo patients.

Estimated % of different categories for IMPS790 (drug = 0)

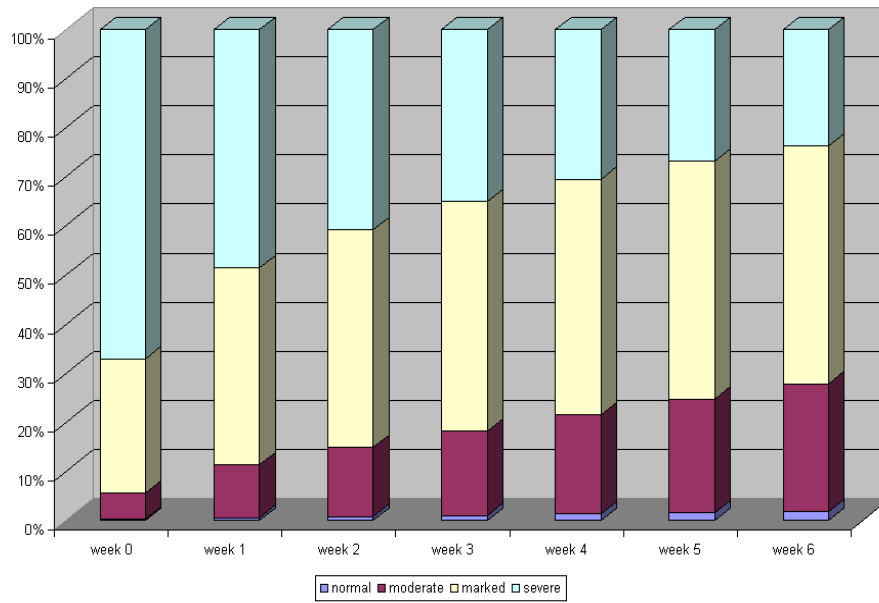


Figure XXX.8: Estimated percentage of patients over time (control group)

Estimated % of different categories for IMPS790 (drug = 1)

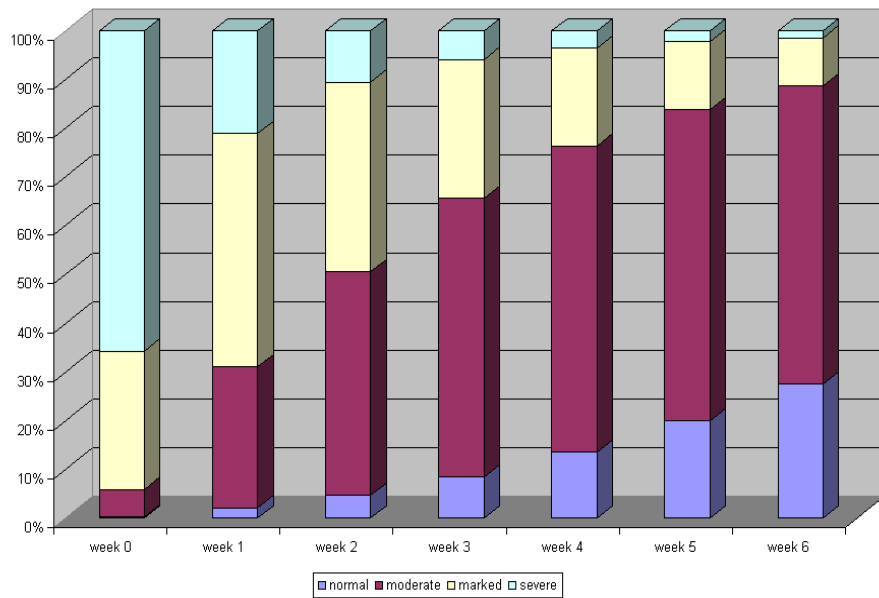


Figure XXX.9: Estimated percentage of patients over time (treatment group)

1.2.3 A 2-level random intercept model and trend model

In this section, we fit a model with random intercept and slope. To do this, the level-1 model is unchanged; only the level-2 model is modified.

1.2.3.1 The model

Level 1 model:

$$y_{ij} = \log\left(\frac{P_{ijc}}{1-P_{ijc}}\right) = \gamma_c - \left[b_{0i} + b_{1i}\text{DRUG}_i + b_{2i}\text{SQRTWEEK}_i + b_{3i}(\text{WSQRT}\times\text{DRUG})_i \right],$$
$$j = 1, \dots, n_i; c = 1, 2, \dots, C - 1$$

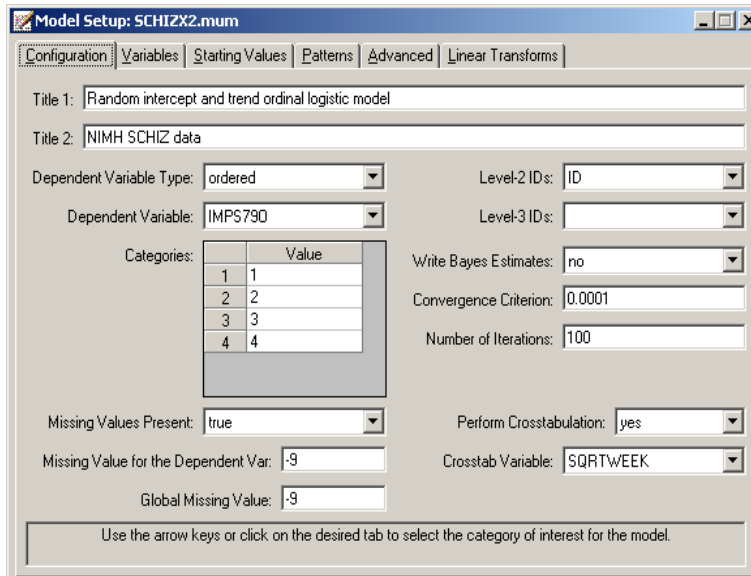
Level 2 model:

$$b_{0i} = \beta_0 + v_{0i}, \quad i = 1, \dots, N$$
$$b_{1i} = \beta_1$$
$$b_{2i} = \beta_2 + v_{2i}$$
$$b_{3i} = \beta_3$$

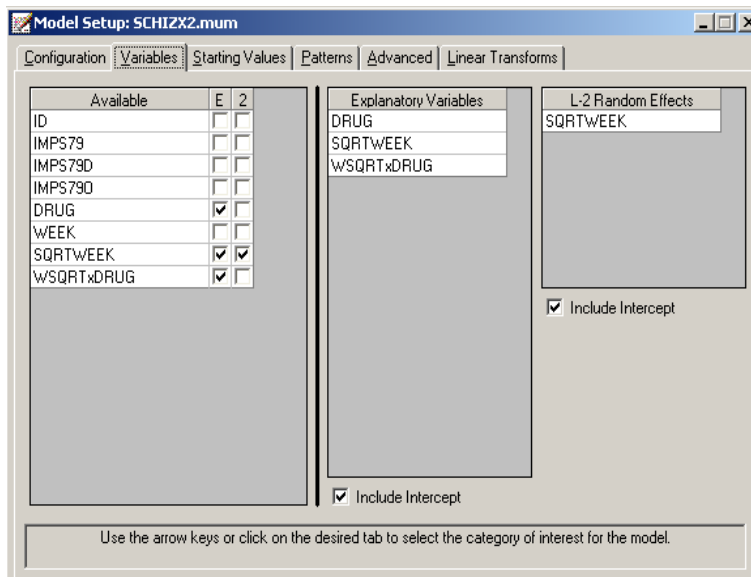
As shown above, the slope of the time variable b_{2i} is now estimated by a level-2 equation containing both a fixed and a random effect. It indicates that patients are now not only assumed to have different intercepts, but may also exhibit different responses to the treatment over time.

1.2.3.2 Setting up the analysis

Use the **File, Open Spreadsheet** option to re-open the previously used spreadsheet **schizx.ss3** from the **Examples\Ordinal** folder. Next, use the **File, Open Existing Model Setup** option to locate and open the syntax file **SCHIZX1.mum**. Click on **File, Save as** to save the model setup in a new file, such as **SCHIZX2.mum**. Next, we change the string in the **Title 1** text box on the **Configuration** screen (optional). Request a crosstabulation of the variable **SQRTWEEK** by the response variable **IMPS790** by selecting the **yes** option from the **Perform Crosstabulation** drop-down list box, followed by the selection of **SQRTWEEK** as the **Crosstab Variable**.



Proceed to the **Variables** tab, and check the **2** check box for SQRTWEEK to select it as a level-2 random variable as shown below.

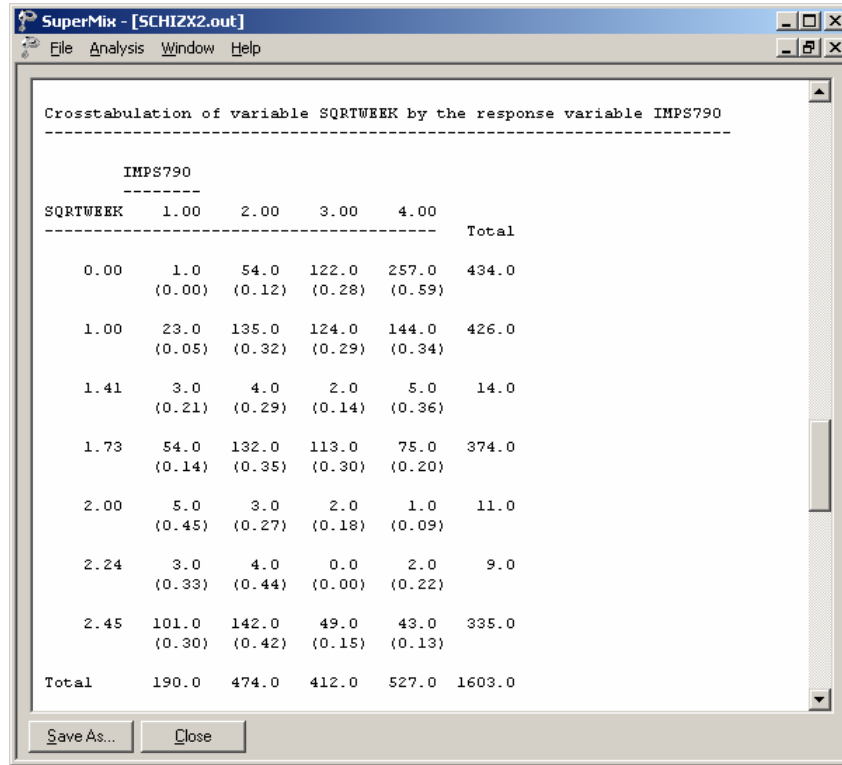


Keep all the other settings unchanged. Save the changes to the file **SCHIZX2.mum** and click the **Analysis, Run** option to produce the output file **SCHIZX2.out**.

1.2.3.3 Discussion of results

Crosstabulation

The following portion of the output is a crosstabulation of the seven distinct values of the variable SQRTWEEK by the four categories of the outcome variable IMPS790. We note that there are relatively few observations for the third, fifth and sixth weeks. For example, for week 5 (SQRTWEEK = 2.24), measurements on only 9 of the 437 patients are available. Looking down the columns (SQRTWEEK) we see the severity of symptoms (IMPS790) declining.



The screenshot shows a window titled "SuperMix - [SCHIZX2.out]" with a menu bar (File, Analysis, Window, Help). The main content area displays a crosstabulation of variable SQRTWEEK by the response variable IMPS790. The table includes counts and percentages for each combination of SQRTWEEK and IMPS790, along with a Total column. The SQRTWEEK values are 0.00, 1.00, 1.41, 1.73, 2.00, 2.24, and 2.45. The IMPS790 values are 1.00, 2.00, 3.00, and 4.00. The Total column shows the sum of counts for each SQRTWEEK value, and the bottom row shows the total counts for each IMPS790 value.

SQRTWEEK	IMPS790				Total
	1.00	2.00	3.00	4.00	
0.00	1.0 (0.00)	54.0 (0.12)	122.0 (0.28)	257.0 (0.59)	434.0
1.00	23.0 (0.05)	135.0 (0.32)	124.0 (0.29)	144.0 (0.34)	426.0
1.41	3.0 (0.21)	4.0 (0.29)	2.0 (0.14)	5.0 (0.36)	14.0
1.73	54.0 (0.14)	132.0 (0.35)	113.0 (0.30)	75.0 (0.20)	374.0
2.00	5.0 (0.45)	3.0 (0.27)	2.0 (0.18)	1.0 (0.09)	11.0
2.24	3.0 (0.33)	4.0 (0.44)	0.0 (0.00)	2.0 (0.22)	9.0
2.45	101.0 (0.30)	142.0 (0.42)	49.0 (0.15)	43.0 (0.13)	335.0
Total	190.0	474.0	412.0	527.0	1603.0

Fixed effect results

The final results after 23 iterations are listed below. While the values of the estimated coefficients differ from those in the random-intercept-only model, the overall picture remains very similar. The decline in severity over time noticed in the crosstabulation is captured by the significant fixed effect coefficient of -0.88295 for SQRTWEEK.

```

schizo2.out
-----
* Final Results - Maximum Marginal Likelihood Estimates *
-----

Total Iterations = 25
Quad Pts per Dim = 25
Log Likelihood = -1662.741
Deviance (-2logL) = 3325.483
Ridge = 0.000

Variable      Estimate      Stand. Error      Z      p-value
-----
intcept      7.32126      0.48075      15.22888      0.00000 (2)
TxDrug       0.05584      0.39907      0.13993      0.88872 (2)
SqrtWeek     -0.88295      0.23461      -3.76353      0.00017 (2)
Tx*SWeek     -1.69416      0.26815      -6.31797      0.00000 (2)

Random effect variance & covariance terms (Cholesky of var-covariance matrix)
intcept      2.64543      0.25859      10.23036      0.00000 (1)
covariance   -0.57054      0.16125      -3.53816      0.00040 (2)
SqrtWeek     1.29774      0.12727      10.19641      0.00000 (1)

Thresholds (for identification: threshold 1 = 0)
2      3.90143      0.21324      18.29581      0.00000 (1)
3      6.50732      0.28996      22.44245      0.00000 (1)

note: (1) = 1-tailed p-value
      (2) = 2-tailed p-value

```

Random effects results

We note from the results above that the estimated coefficient for the random SQRTWEEK slope is highly significant, indicating that patients not only start at different points but follow different paths during the treatment period. At the end of the output file, the calculation of the random effects variance-covariance matrix is given.

```

schizo2.out
-----
Calculation of the random effects variance-covariance matrix
-----
intcept  variance = (2.645 * 2.645) = 6.998
          covariance = (2.645 * -.571) = *****
SqrtWeek variance = (-.571 * -.571) + (1.298 * 1.298) = 2.010

Covariance expressed as a correlation = -.402
-----
Save As...  Close

```

1.2.3.4 Interpreting the output

Estimated outcomes for groups: unit-specific results

To evaluate the expected effect of the treatment group and the square root of time of treatment, while allowing for the interaction between treatment and the square root of time, we use the expression below:

$$\log\left(\frac{\hat{P}_{ijc}}{1-\hat{P}_{ijc}}\right) = \hat{\gamma}_c - \left[\hat{b}_{0i} + \hat{b}_{1i} \text{DRUG}_i + \hat{b}_{2i} \text{SQRTWEEK}_i + \hat{b}_{3i} (\text{WSQRT} \times \text{DRUG})_i \right]$$

so that

$$\begin{aligned} \hat{\eta}_{ijc} = & \hat{\gamma}_c - 7.3086 - 0.1109 \times \text{DRUG}_i + 0.87523 \times \text{SQRTWEEK}_i \\ & + 1.72433 \times (\text{WSQRT} \times \text{DRUG})_i \end{aligned}$$

As illustrated in the previous example, by substituting the values for DRUG, SQRTWEEK and WSQRTxDRUG, the results shown in Table XXX8 below can be obtained.

TABLE XXX8: Estimated unit-specific results for random intercept & slope model

severity	Placebo patients (drug = 0)				Drug patients (drug = 1)			
	normal	moderate	marked	severe	normal	moderate	marked	Severe
week 0	0.07%	3.17%	28.19%	68.57%	0.06%	2.85%	26.18%	70.91%
week 1	0.16%	7.28%	44.93%	47.63%	0.80%	27.94%	55.92%	15.34%
week 2	0.23%	10.12%	50.89%	38.76%	2.31%	51.90%	39.98%	5.81%
week 3	0.30%	12.93%	54.37%	32.40%	5.13%	67.87%	24.36%	2.63%
week 4	0.38%	15.78%	56.35%	27.48%	9.79%	74.65%	14.23%	1.33%
week 5	0.47%	18.69%	57.28%	23.56%	16.70%	74.23%	8.35%	0.72%
week 6	0.57%	21.65%	57.41%	20.37%	25.89%	68.70%	5.00%	0.42%

We can again represent the results from the above table graphically, as shown in the following two graphs. The graphs tell us the same story as the previous model: patients from the treatment group showed more improvement over time than patients from the control group. While a very small proportion of treatment patients were still diagnosed as being severely ill at the end of the treatment period (0.42% according to table XXX8), 20% of the control group were still classified as being severely ill by week 6.

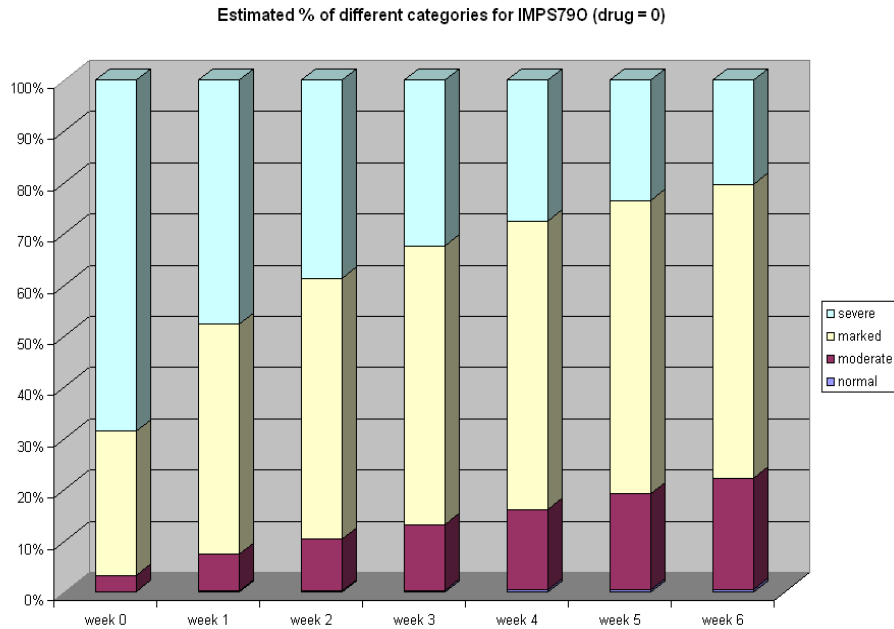


Figure XXX.10: Estimated percentage of patients over time (treatment group)

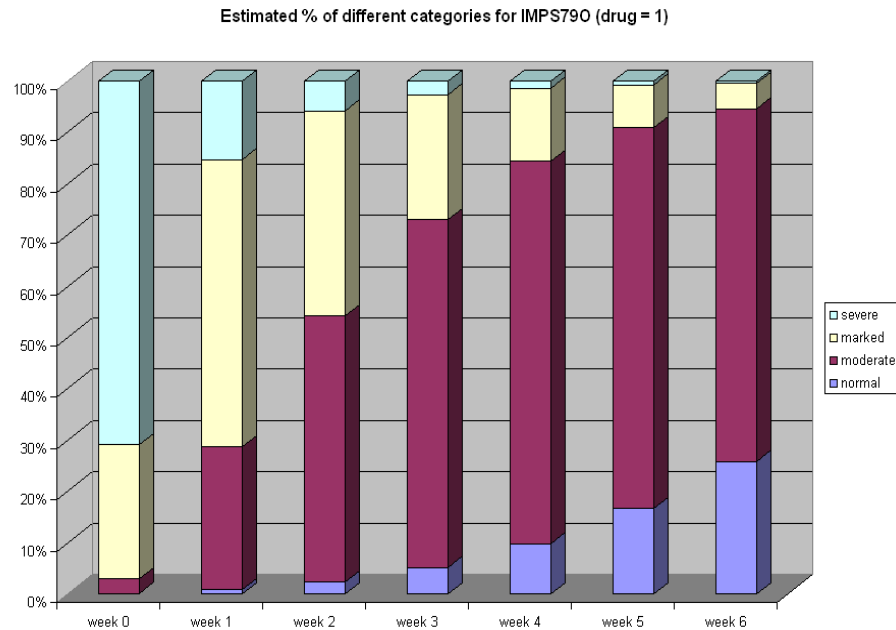


Figure XXX.11: Estimated percentage of patients over time (treatment group)

Estimated time trend variance

When we consider the heterogeneity in responses across time, we notice that the estimated variance in the time trend is $\sigma_{v_1}^2 = (1.29774)^2 + (-0.57054)^2 = 2.0096$. The estimates for the time trends are -0.88295 for SQRTWEEK and -1.69416 for WSQRTxDR respectively. Thus the estimated trends for the placebo and drug groups are -0.88295 and $-0.88295 - 1.69416 = -2.57711$. Thus the 95% confidence interval of the time trend for the placebo group is $-0.88295 \pm (1.96 \times \sqrt{2.0096}) = (-3.6615, 1.896)$. Similarly, the confidence interval for the drug group is $(-5.3556, 0.2014)$. Notice that both intervals are fairly large and include negative and positive slopes, which reflects the wide heterogeneity in trends. The estimated correlation value is -0.402 , which is moderately large. This indicates that the patients who are initially less severely ill improve at a smaller rate. The more severely ill patients improve at a greater rate.