# More Mixed-Effects Models for Ordinal & Nominal Data

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## Proportional and Non-proportional Odds

*Proportional Odds model*

$$\log\left[\frac{P(\mathsf{y}_{ij} \leq c)}{1 - P(\mathsf{y}_{ij} \leq c)}\right] = \gamma_c - \left[\boldsymbol{x}'_{ij}\boldsymbol{\beta} + \boldsymbol{z}'_{ij}\boldsymbol{\upsilon}_i\right]$$

$$\text{with } \boldsymbol{\upsilon}_i \sim N(\mathbf{0}, \boldsymbol{T}\boldsymbol{T}' = \boldsymbol{\Sigma}_\upsilon)$$

$$= \gamma_c - \left[\boldsymbol{x}'_{ij}\boldsymbol{\beta} + \boldsymbol{z}'_{ij}\boldsymbol{T}\boldsymbol{\theta}_i\right] \;\; \text{with } \boldsymbol{\theta}_i \sim N(\mathbf{0}, \boldsymbol{I})$$

- relationship between the explanatory variables and the cumulative logits does not depend on $c$
- effects of $\boldsymbol{x}$ variables DO NOT vary across the $C - 1$ cumulative logits

Hedeker & Mermelstein (1998, *Mult Behav Res*) extension:

$$\log\left[\frac{P(\mathsf{y}_{ij} \le c)}{1 - P(\mathsf{y}_{ij} \le c)}\right] = \gamma_{c(0)} - [\boldsymbol{u}'_{ij}\boldsymbol{\gamma}_c + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \boldsymbol{z}'_{ij}\boldsymbol{T}\boldsymbol{\theta}_i]$$

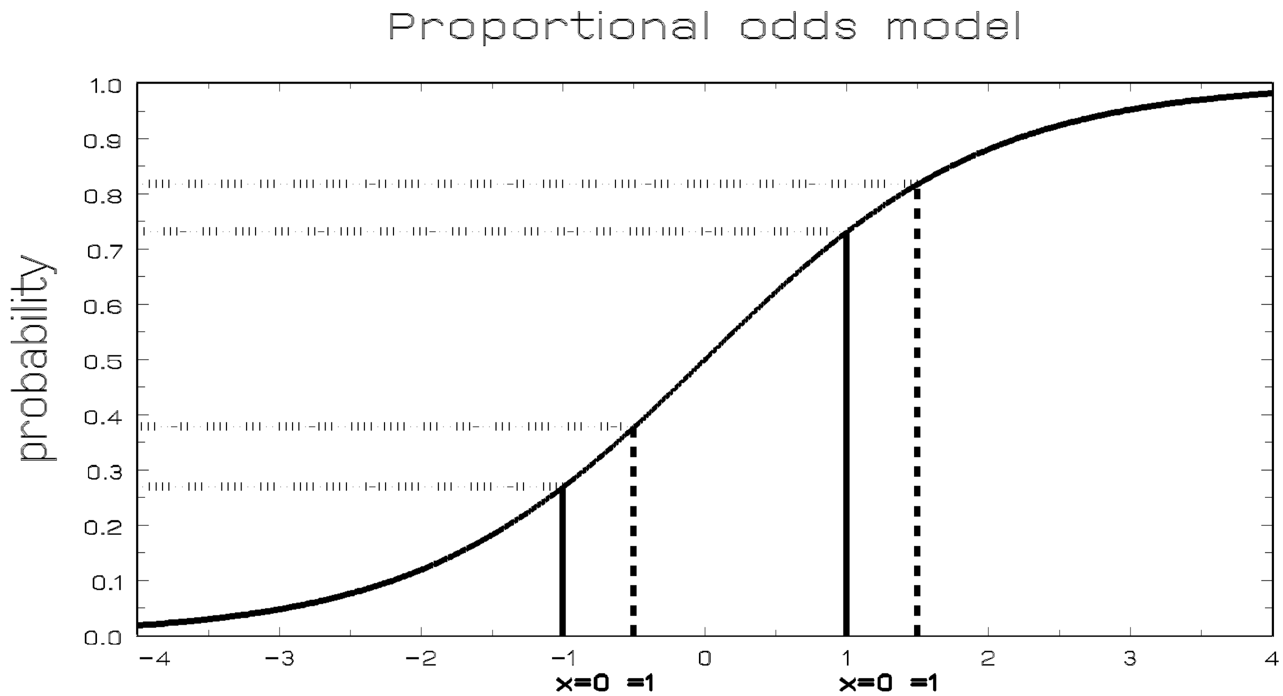
$\boldsymbol{u}_{ij} = h \times 1$ vector for the set of $h$ covariates for which proportional odds is not assumed

- effects of $\boldsymbol{u}$ variables DO vary across the $C - 1$ cumulative logits
- more flexible model for ordinal response relations

**Proportional Odds Assumption**: covariate effects are the same across all cumulative logits

| group | *Response* Absent | Mild | Severe | total |
|---|---|---|---|---|
| control | 27 | 46 | 27 | 100 |
| cumulative odds | $\frac{27}{73} = .37$ | $\frac{73}{27} = 2.7$ | | |
| *logit* | *-1* | *1* | | |
| treatment | 38 | 44 | 18 | 100 |
| cumulative odds | $\frac{38}{62} = .61$ | $\frac{82}{18} = 4.6$ | | |
| *logit* | *-.5* | *1.5* | | |

$\Rightarrow$ *group difference = .5 for both cumulative logits*




$$\log\left[\frac{P(\mathsf{y}_{ij} \le 1)}{1 - P(\mathsf{y}_{ij} \le 1)}\right] = \log\left[\frac{P(\mathsf{y}_{ij} = 1)}{P(\mathsf{y}_{ij} = 2 \text{ or } 3)}\right] = 0 - [\beta_0 + x\,\beta_1]$$

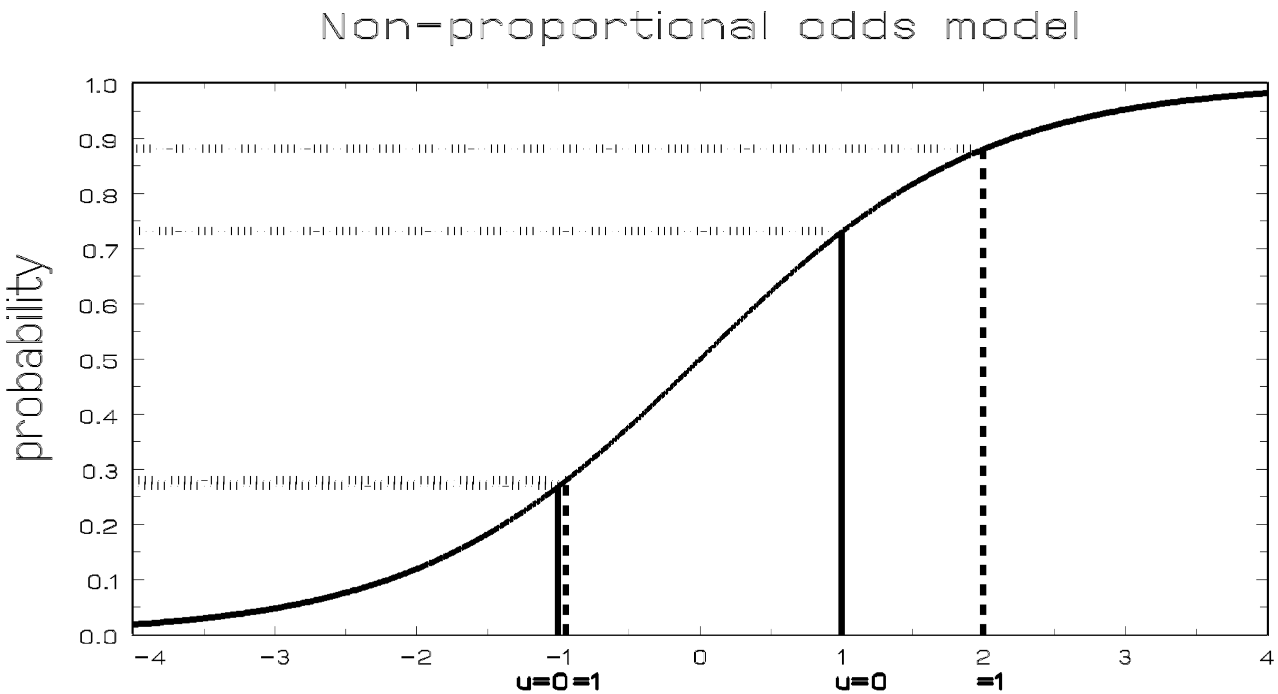
$$\log\left[\frac{P(\mathsf{y}_{ij} \le 2)}{1 - P(\mathsf{y}_{ij} \le 2)}\right] = \log\left[\frac{P(\mathsf{y}_{ij} = 1 \text{ or } 2)}{P(\mathsf{y}_{ij} = 3)}\right] = \gamma_2 - [\beta_0 + x\,\beta_1]$$

$\beta_0 = 1\,, \quad \gamma_2 = 2\,, \quad \beta_1 = -.5$
(covariate effect is same for both cumulative logits)

**Non-Proportional Odds**: covariate effects vary across the cumulative logits

| group | *Response* Absent | Mild | Severe | total |
|---|---|---|---|---|
| control | 27 | 46 | 27 | 100 |
| cumulative odds | $\frac{27}{73} = .37$ | $\frac{73}{27} = 2.7$ | | |
| *logit* | *-1* | *1* | | |
| treatment | 28 | 60 | 12 | 100 |
| cumulative odds | $\frac{28}{72} = .39$ | $\frac{88}{12} = 7.3$ | | |
| *logit* | *-.95* | *2* | | |

$\Rightarrow$ *UNEQUAL group difference across cumulative logits*




$$\log\left[\frac{P(\mathsf{y}_{ij} \le 1)}{1 - P(\mathsf{y}_{ij} \le 1)}\right] = \log\left[\frac{P(\mathsf{y}_{ij} = 1)}{P(\mathsf{y}_{ij} = 2 \text{ or } 3)}\right] = 0 - [\beta_0 + u\,\gamma_1]$$

$$\log\left[\frac{P(\mathsf{y}_{ij} \le 2)}{1 - P(\mathsf{y}_{ij} \le 2)}\right] = \log\left[\frac{P(\mathsf{y}_{ij} = 1 \text{ or } 2)}{P(\mathsf{y}_{ij} = 3)}\right] = \gamma_{2\,(0)} - [\beta_0 + u\,\gamma_2]$$

$\beta_0 = 1\,, \quad \gamma_{2(0)} = 2\,, \quad \gamma_1 = -.05\,, \quad \gamma_2 = -.1$
(covariate effect varies across the cumulative logits)

## Mixed-effects Multinomial Logistic Regression Model for Nominal Responses (Hedeker, 2003)

$\mathsf{y}_{ij}$ = nominal response of level-2 unit $i$ and level-1 unit $j$

Which member of The Polkaholics is your favorite?
(asked before, during, and after a show)

# Mixed-effects Multinomial Logistic Regression Model

$$\log \frac{p_{ijc}}{p_{ij1}} = \boldsymbol{u}'_{ij}\boldsymbol{\gamma}_c + \boldsymbol{z}'_{ij}\boldsymbol{T}_c\,\boldsymbol{\theta}_i \qquad c = 2, 3, \ldots C$$

- $C - 1$ contrasts to reference cell ($c = 1$)
- regression effects $\boldsymbol{\gamma}_c$ vary across contrasts
- random-effects variance terms $\boldsymbol{T}_c$ vary across contrasts

*For example, with C = 3*

| contrast | ordinal | nominal |
|---|---|---|
| *c1* | 2 & 3 vs 1 | 2 vs 1 |
| *c2* | 3 vs 1 & 2 | 3 vs 1 |

## Subject-specific probabilities

- estimated logits ($\hat{\text{z}}$) given $\boldsymbol{\theta}_i$
  - ordinal $\hat{\text{z}}_{ijc}(\boldsymbol{\theta}_i) = \hat{\gamma}_{c(0)} - \left[\boldsymbol{u}'_{ij}\hat{\boldsymbol{\gamma}}_c + \boldsymbol{x}'_{ij}\hat{\boldsymbol{\beta}} + \boldsymbol{z}'_{ij}\hat{\boldsymbol{T}}\boldsymbol{\theta}_i\right]$
  - nominal $\hat{\text{z}}_{ijc}(\boldsymbol{\theta}_i) = \boldsymbol{u}'_{ij}\hat{\boldsymbol{\gamma}}_c + \boldsymbol{z}'_{ij}\hat{\boldsymbol{T}}_c\,\boldsymbol{\theta}_i$

- estimated probabilities
  - ordinal $\hat{p}_{ijc}(\boldsymbol{\theta}_i) = \frac{1}{1+\exp(-\hat{\text{z}}_{ijc}(\boldsymbol{\theta}_i))} - \frac{1}{1+\exp(-\hat{\text{z}}_{ij,c-1}(\boldsymbol{\theta}_i))}$
  - nominal $\hat{p}_{ijc}(\boldsymbol{\theta}_i) = \frac{\exp(\hat{\text{z}}_{ijc}(\boldsymbol{\theta}_i))}{\Sigma_{h=1}^{C}\exp(\hat{\text{z}}_{ijh}(\boldsymbol{\theta}_i))}$ with $\text{z}_{ij1} = 0$

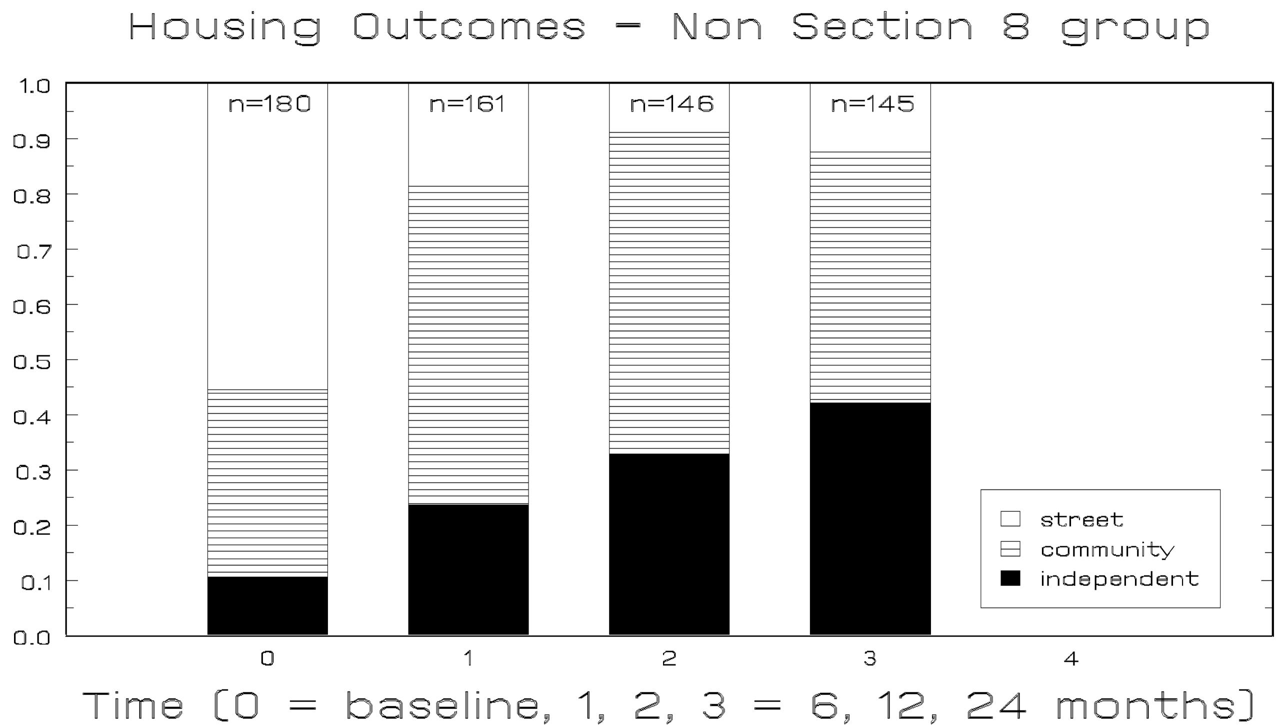
## Marginal probabilities

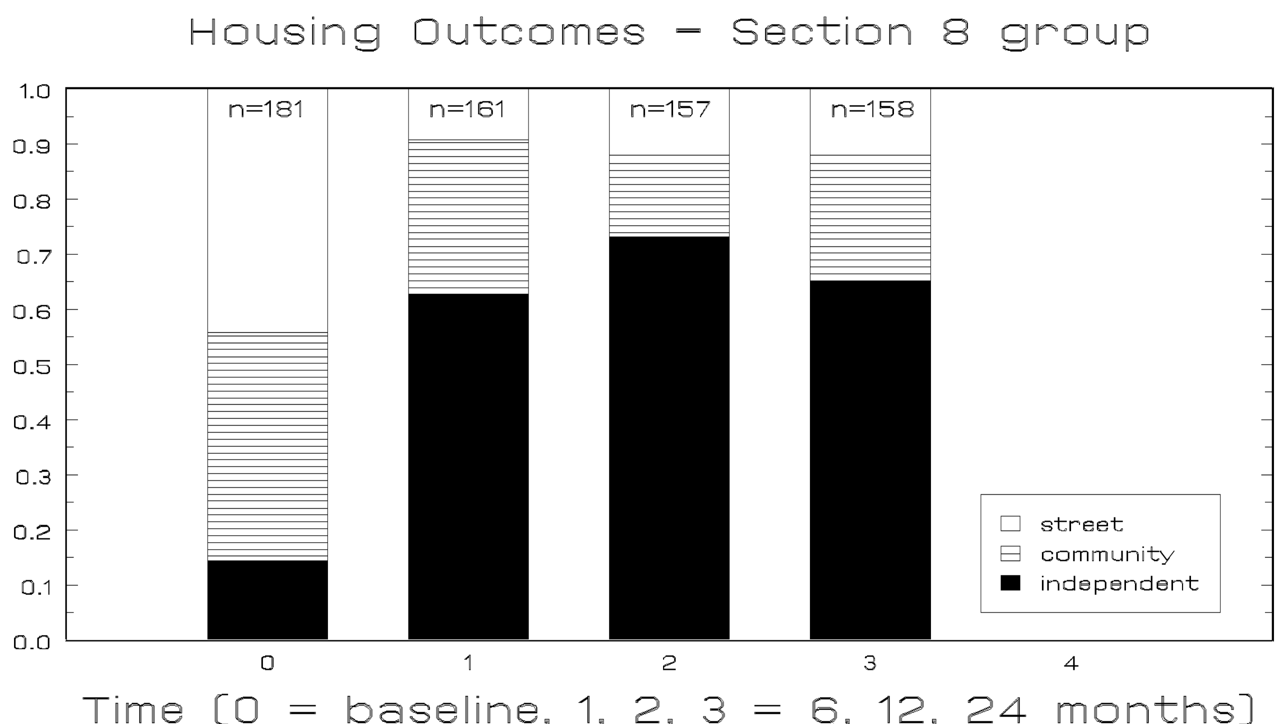
$$\hat{p}_{ijc} = \int_{\boldsymbol{\theta}} \hat{p}_{ijc}(\boldsymbol{\theta}_i)\, g(\boldsymbol{\theta})\, d\boldsymbol{\theta} \approx \sum_{q=1}^{Q} \hat{p}_{ijc}(\boldsymbol{B}_q)\, W(\boldsymbol{B}_q)$$

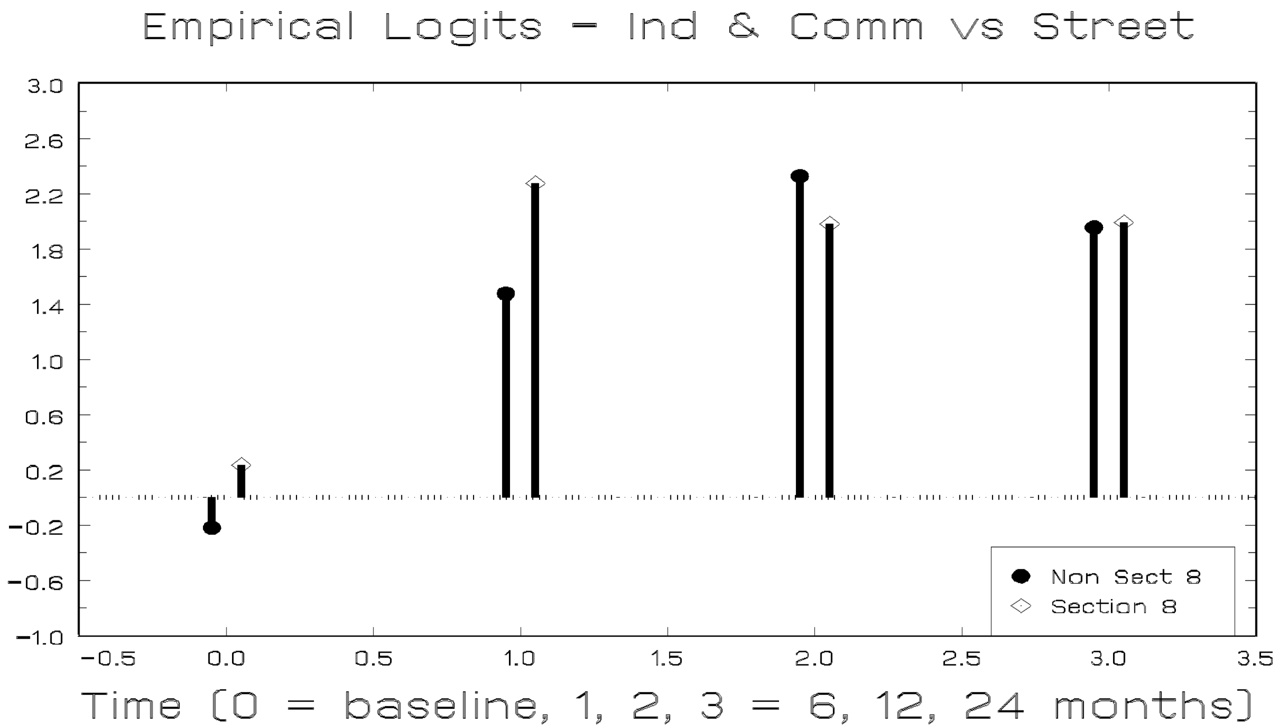
## **San Diego Homeless Research Project** (Hough)

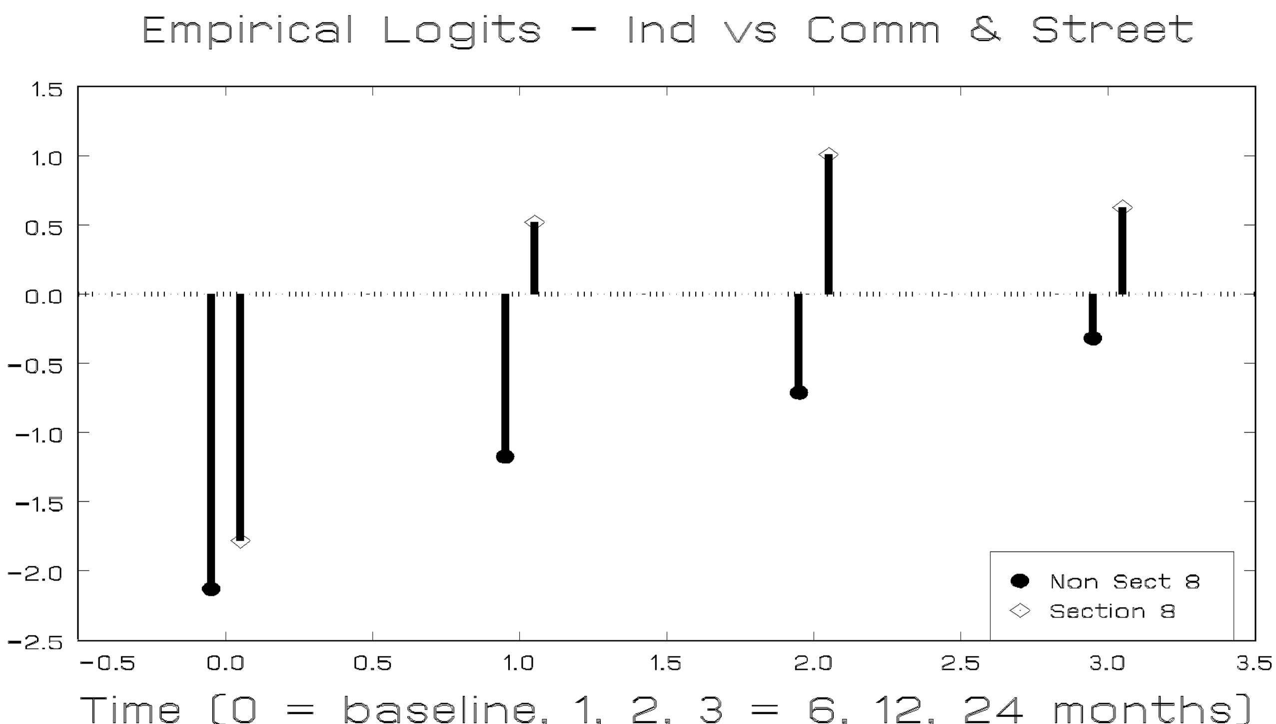
- 361 mentally ill subjects who were
  - homeless or
  - at very high risk of becoming homeless
- 2 conditions: HUD Section 8 rental certificates (yes/no)
- baseline and 6, 12, and 24 month follow-ups
- Categorical outcome: housing status
  - streets / shelters ($\mathsf{y} = 1$)
  - community / institutions ($\mathsf{y} = 2$)
  - independent ($\mathsf{y} = 3$)

*Question:* Do Section 8 certificates influence housing status across time?


Housing Outcomes - Non Section 8 group
n=180
n=161
n=146
n=145
street
community
independent
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)


Housing Outcomes - Section 8 group
n=181
n=161
n=157
n=158
street
community
independent
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)


Empirical Logits - Ind & Comm vs Street
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)
Non Sect 8
Section 8


Empirical Logits - Ind vs Comm & Street
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)
Non Sect 8
Section 8

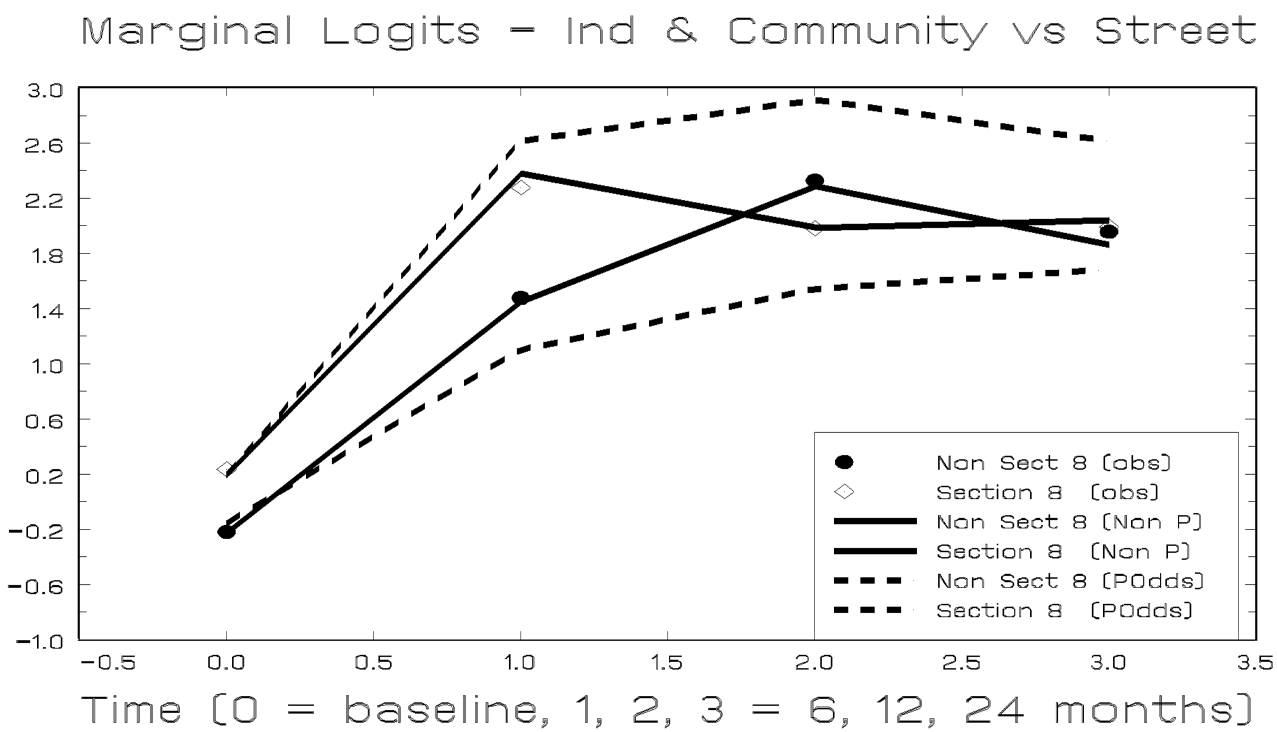
Housing status across time: 1289 observations within 361 subjects
Ordinal Mixed Regression Model estimates and standard errors (se)

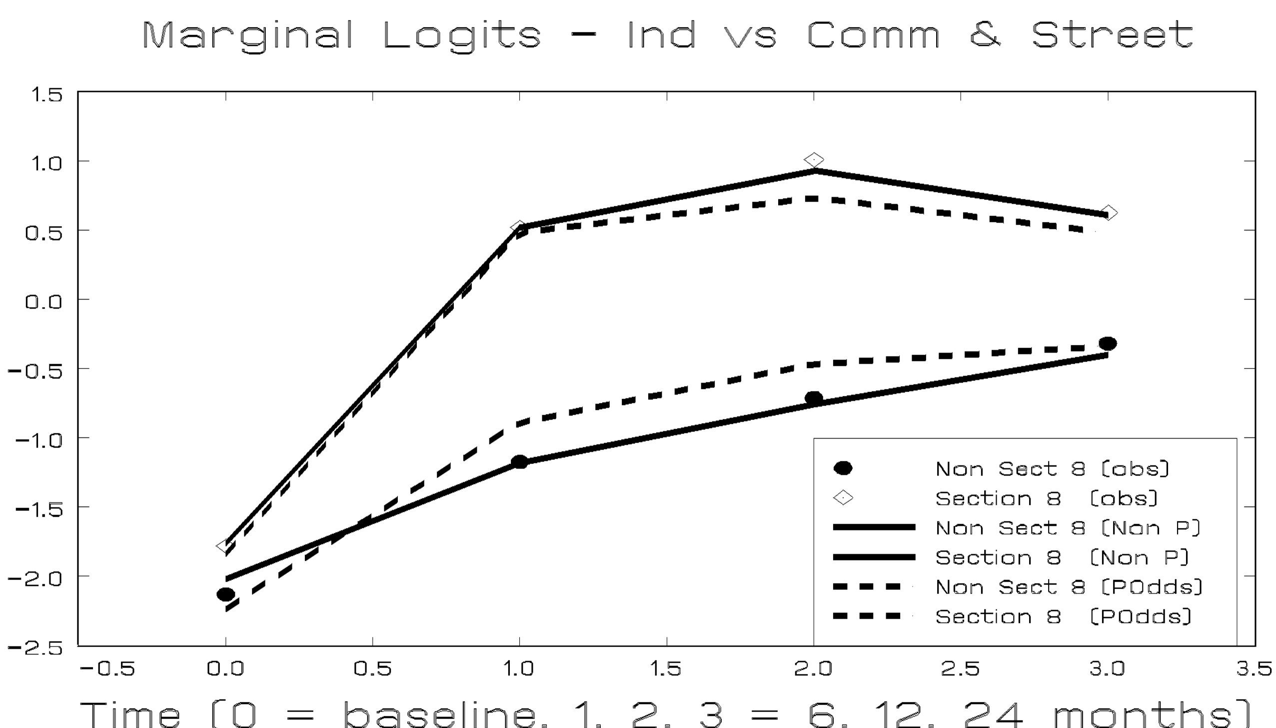
| | Proportional Odds Model | | Non-Proportional Odds | | | |
|---|---|---|---|---|---|---|
| | | | Non-street[1] | | Independent[2] | |
| term | estimate | se | estimate | se | estimate | se |
| intercept | - .220 | .203 | - .322 | .218 | | |
| threshold | **2.744** | .110 | | | **2.377** | .279 |
| t1 (6 month) | **1.736** | .233 | **2.297** | .298 | **1.079** | .358 |
| t2 (12 month) | **2.315** | .268 | **3.345** | .450 | **1.645** | .336 |
| t3 (24 month) | **2.499** | .247 | **2.821** | .369 | **2.145** | .339 |
| section 8 (y=1) | *.497* | .280 | *.592* | .305 | .323 | .401 |
| section 8 by t1 | **1.408** | .334 | .566 | .478 | **2.023** | .478 |
| section 8 by t2 | **1.173** | .360 | -.958 | .582 | **2.016** | .466 |
| section 8 by t3 | *.638* | .331 | -.366 | .506 | **1.073** | .472 |
| subject sd | **1.459** | .106 | **1.457** | .112 | ($ICC \approx .4$) | |
| $-2 \log L$ | 2274.39 | | 2222.25 | | ($\chi^2_7 = 52.14$) | |

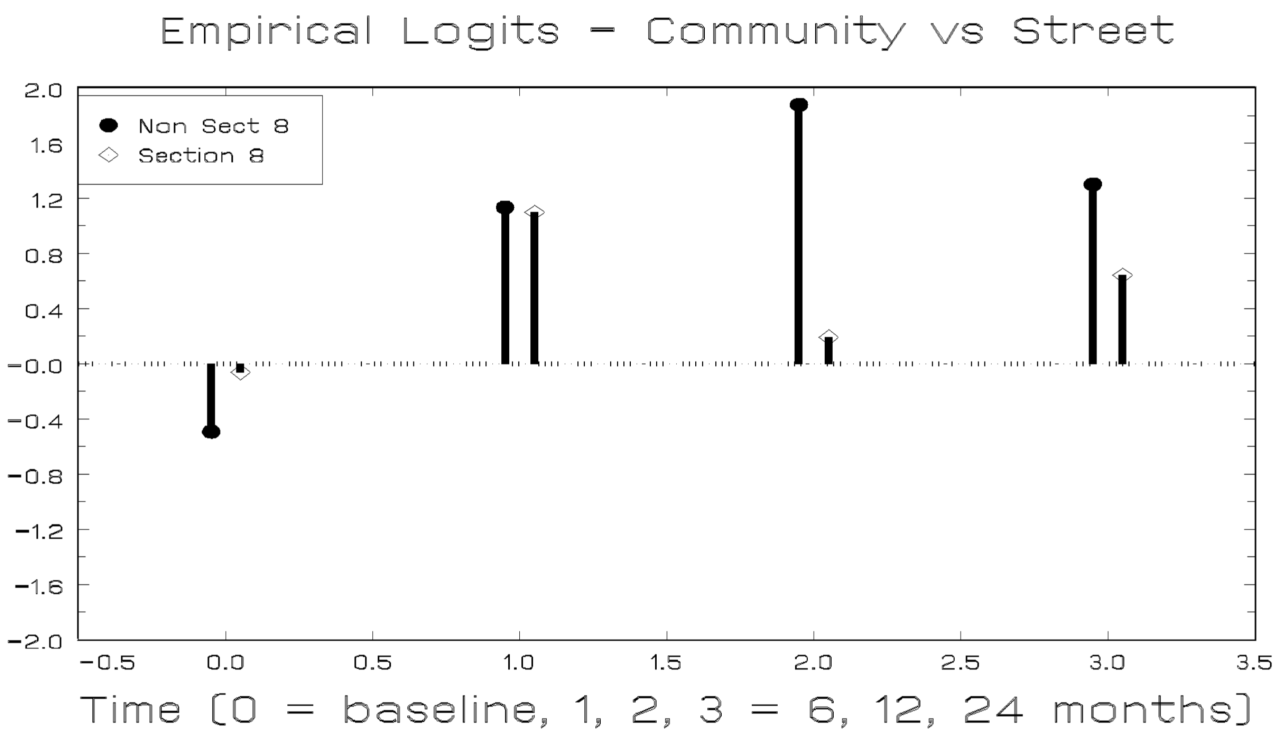
**bold** indicates $p < .05$ *italic* indicates $.05 < p < .10$

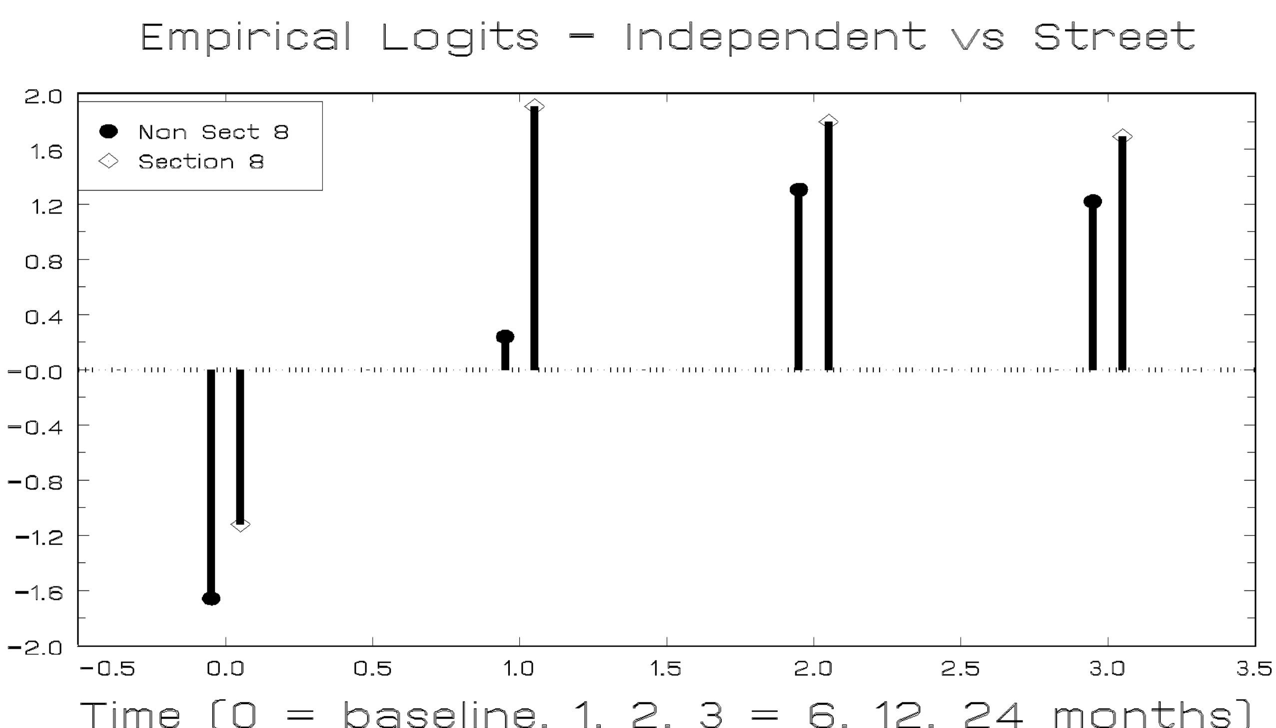
[1] = independent + community vs street

[2] = independent vs community + street


Marginal Logits - Ind & Community vs Street
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
-0.2
-0.6
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Non Sect 8 (obs)
Section 8 (obs)
Non Sect 8 (Non P)
Section 8 (Non P)
Non Sect 8 (POdds)
Section 8 (POdds)
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)


Marginal Logits - Ind vs Comm & Street
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Non Sect 8 (obs)
Section 8 (obs)
Non Sect 8 (Non P)
Section 8 (Non P)
Non Sect 8 (POdds)
Section 8 (POdds)
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)

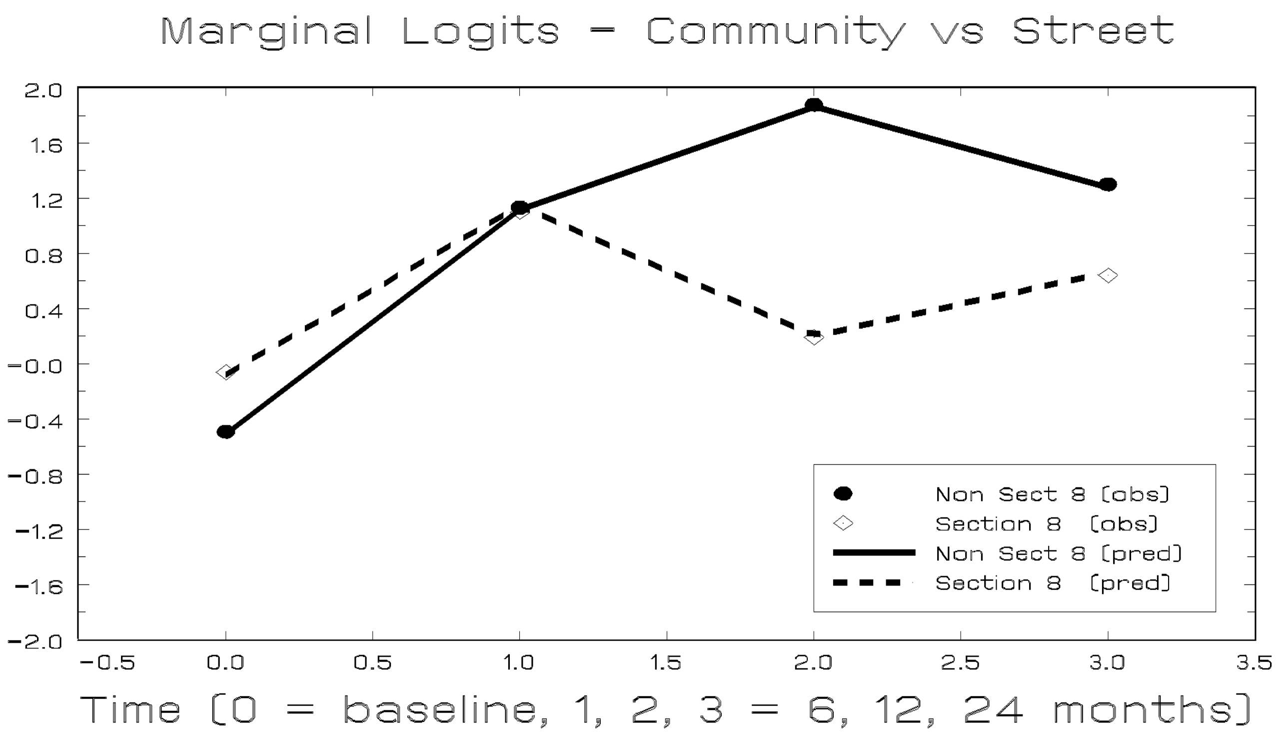

Empirical Logits - Community vs Street
Non Sect 8
Section 8
2.0
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
-1.6
-2.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)

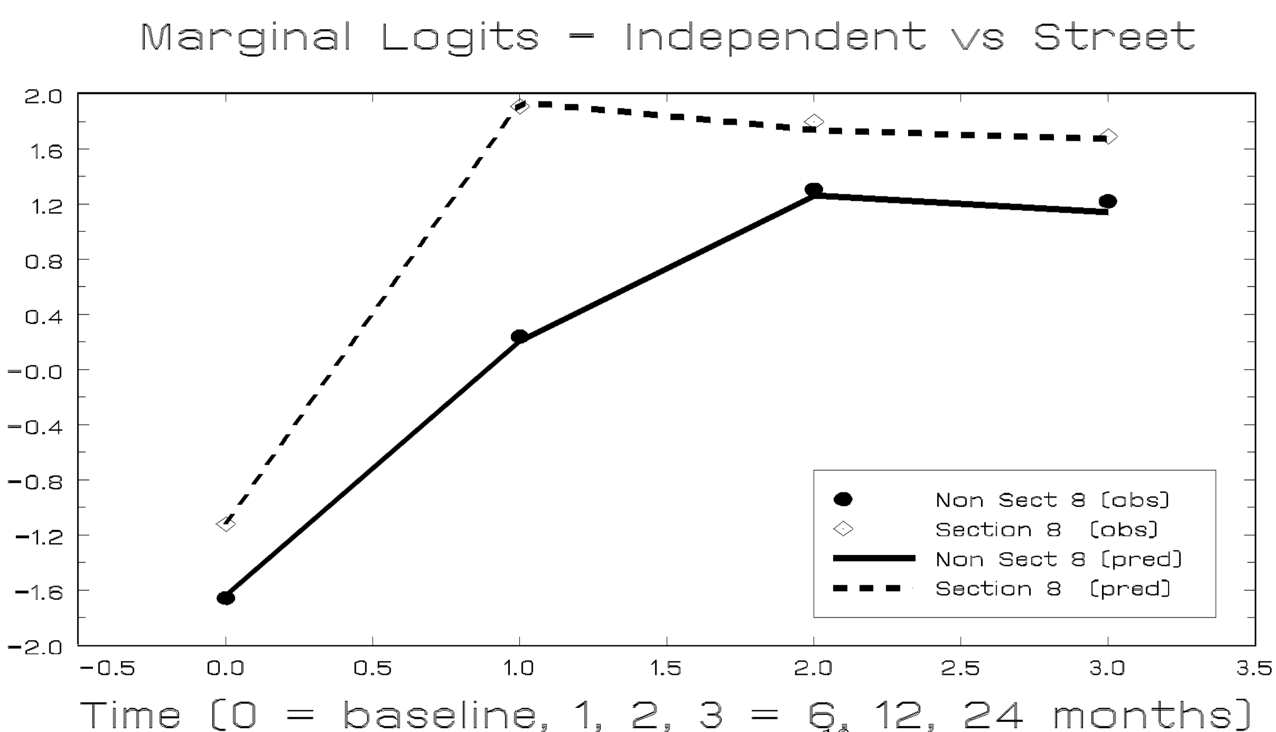

Empirical Logits - Independent vs Street
Non Sect 8
Section 8
2.0
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
-1.6
-2.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)

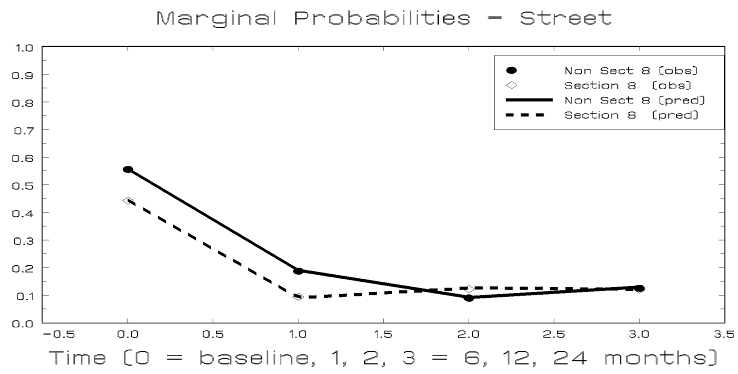
Housing status across time: 1289 observations within 361 subjects
Nominal Mixed Regression Model estimates & standard errors (se)

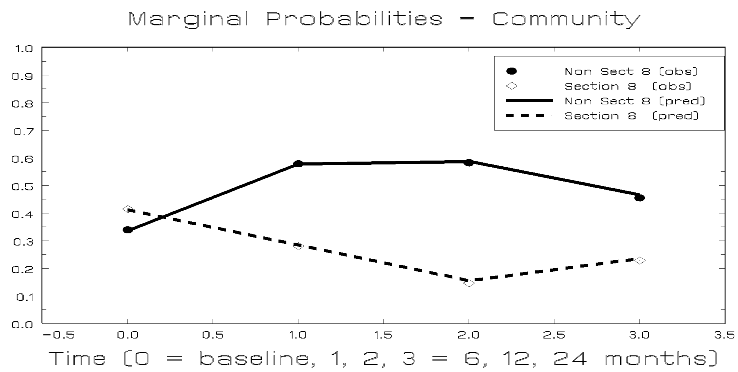
| | Community vs Street | | Independent vs Street | |
|---|---|---|---|---|
| term | estimate | se | estimate | se |
| intercept | - .452 | .192 | -2.675 | .367 |
| t1 (6 month) | **1.942** | .312 | **2.682** | .425 |
| t2 (12 month) | **2.820** | .466 | **4.088** | .559 |
| t3 (24 month) | **2.259** | .378 | **4.099** | .469 |
| section 8 (y=1) | *.521* | .268 | .781 | .491 |
| section 8 by t1 | -.135 | .490 | **2.003** | .614 |
| section 8 by t2 | **-1.917** | .611 | .548 | .694 |
| section 8 by t3 | -*.952* | .535 | .304 | .615 |
| subject sd | **.871** | .138 | **2.334** | .196 |
| $-2 \log L$ | 2218.73 | | | |

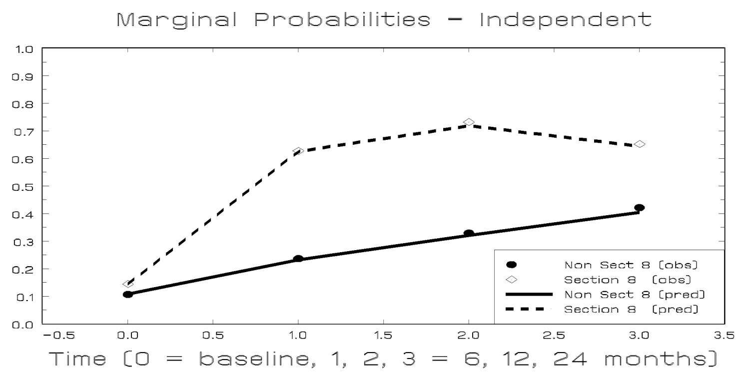
**bold** indicates $p < .05$ *italic* indicates $.05 < p < .10$


Marginal Logits - Community vs Street
2.0
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
-1.6
-2.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Non Sect 8 (obs)
Section 8 (obs)
Non Sect 8 (pred)
Section 8 (pred)
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)


Marginal Logits - Independent vs Street
2.0
1.6
1.2
0.8
0.4
-0.0
-0.4
-0.8
-1.2
-1.6
-2.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Non Sect 8 (obs)
Section 8 (obs)
Non Sect 8 (pred)
Section 8 (pred)
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)


Marginal Probabilities - Street
Non Sect 8 (obs)
Section 8 (obs)
Non Sect 8 (pred)
Section 8 (pred)
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)


Marginal Probabilities - Community
Non Sect 8 (obs)
Section 8 (obs)
Non Sect 8 (pred)
Section 8 (pred)
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)


Marginal Probabilities - Independent
Non Sect 8 (obs)
Section 8 (obs)
Non Sect 8 (pred)
Section 8 (pred)
Time (0 = baseline, 1, 2, 3 = 6, 12, 24 months)

**SAS IML code:** computing marginal probabilities - ordinal models

```
TITLE1 'San Diego Homeless Data - Estimated Marginal Probabilities';
PROC IML;
/* Results from MIXOR analysis:  proportional odds model */;

 x0  =  {  0 0 0 0 0 0 0,
           0 1 0 0 0 0 0,
           0 0 1 0 0 0 0,
           0 0 0 1 0 0 0};
 x1  =  {  1 0 0 0 0 0 0,
           1 1 0 0 1 0 0,
           1 0 1 0 0 1 0,
           1 0 0 1 0 0 1};

 int     =  {-.220};
 sd      =  {1.459};
 beta    =  {.497, 1.736, 2.315, 2.499, 1.408, 1.173, .638};
 thresh  =  {2.744};
```

```
/* number of quadrature points, quadrature nodes & weights */
nq = 10;
bq = { -4.85946282833231, -3.58182348355193, -2.48432584163895,
       -1.46598909439116, -0.48493570751550,  0.48493570751550,
        1.46598909439116,  2.48432584163895,  3.58182348355193,
        4.85946282833231};
wq = { 0.00000431065265, 0.00075807095698, 0.01911158107317,
       0.13548370704150, 0.34464234526294, 0.34464234526294,
       0.13548370704150, 0.01911158107317, 0.00075807095698,
       0.00000431065265};

/* initialize to zero */
 grp0a  =  J(4,1,0);
 grp0b  =  J(4,1,0);
 grp1a  =  J(4,1,0);
 grp1b  =  J(4,1,0);
```

```
DO q = 1 to nq;

    za0 = 0 - (int + x0*beta + sd*bq[q]);
    zb0 = thresh - (int + x0*beta + sd*bq[q]);
    za1 = 0 - (int + x1*beta + sd*bq[q]);
    zb1 = thresh - (int + x1*beta + sd*bq[q]);

    grp0a = grp0a + ( 1 / ( 1 + EXP(0 - za0)))*wq[q];
    grp0b = grp0b + ( 1 / ( 1 + EXP(0 - zb0)))*wq[q];

    grp1a = grp1a + ( 1 / ( 1 + EXP(0 - za1)))*wq[q];
    grp1b = grp1b + ( 1 / ( 1 + EXP(0 - zb1)))*wq[q];

END;

print 'Proportional odds model';
print 'Quadrature method - 10 points';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (1-grp1c) [FORMAT=8.4];
```

```
/* Approximate Marginalization Method - no quadrature */;

 pi    =  3.141592654;
 nt    =  4;
 ivec  =  J(nt,1,1);
 evec  =  (15/16)**2 * (pi**2)/3 * ivec;

/* nt by nt matrix with evec on the diagonal and zeros elsewhere */;
emat = diag(evec);

/* variance-covariance matrix of underlying latent variable */;
vary = ivec * sd * T(sd) * T(ivec) + emat;

sdy = sqrt(vecdiag(vary) / vecdiag(emat));

za0 = (0 - (int + x0*beta)) / sdy ;
zb0 = (thresh - (int + x0*beta)) / sdy;
za1 = (0 - (int + x1*beta)) / sdy;
zb1 = (thresh - (int + x1*beta)) / sdy;

grp0a = 1 / ( 1 + EXP(0 - za0));
grp0b = 1 / ( 1 + EXP(0 - zb0));
grp1a = 1 / ( 1 + EXP(0 - za1));
grp1b = 1 / ( 1 + EXP(0 - zb1));
```

```
print 'Proportional odds model';
print 'Approximate Marginalization Method';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (1-grp1c) [FORMAT=8.4];
```

```
/* Non-Proportional Odds Model */;

 int    =  {-.322};
 sd     =  {1.457};
 gam1   =  {.592, 2.297, 3.345, 2.821, .566, -.958, -.366};
 gam2   =  {.323, 1.079, 1.645, 2.145, 2.023, 2.016, 1.073};
 thresh =  {2.377};

/* initialize to zero */
 grp0a  =  J(4,1,0);
 grp0b  =  J(4,1,0);
 grp1a  =  J(4,1,0);
 grp1b  =  J(4,1,0);
```

```
DO q = 1 to nq;

    za0 = 0 - (int + x0*gam1 + sd*bq[q]);
    zb0 = thresh - (int + x0*gam2 + sd*bq[q]);
    za1 = 0 - (int + x1*gam1 + sd*bq[q]);
    zb1 = thresh - (int + x1*gam2 + sd*bq[q]);

    grp0a = grp0a + ( 1 / ( 1 + EXP(0 - za0)))*wq[q];
    grp0b = grp0b + ( 1 / ( 1 + EXP(0 - zb0)))*wq[q];

    grp1a = grp1a + ( 1 / ( 1 + EXP(0 - za1)))*wq[q];
    grp1b = grp1b + ( 1 / ( 1 + EXP(0 - zb1)))*wq[q];

END;

print 'Non-proportional odds model';
print 'Quadrature method - 10 points';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (1-grp1c) [FORMAT=8.4];
```

**SAS IML code:** computing marginal probabilities - nominal model

```
TITLE1 'San Diego Homeless Data - Estimated Marginal Probabilities';
PROC IML;
/* Results from MIXNO analysis */;

 u0  =  {  0 0 0 0 0 0 0,
           0 1 0 0 0 0 0,
           0 0 1 0 0 0 0,
           0 0 0 1 0 0 0};
 u1  =  {  1 0 0 0 0 0 0,
           1 1 0 0 1 0 0,
           1 0 1 0 0 1 0,
           1 0 0 1 0 0 1};

 inta  =  {-.452};
 sda   =  {.871};
 gama  =  {.521, 1.942, 2.820, 2.259, -.135, -1.917, -.952};

 intb  =  {-2.675};
 sdb   =  { 2.334};
 gamb  =  {.781, 2.682, 4.088, 4.100, 2.003, .548, .304};
```

```
/* number of quadrature points, quadrature nodes & weights */
nq = 10;
bq = { -4.85946282833231, -3.58182348355193, -2.48432584163895,
       -1.46598909439116, -0.48493570751550,  0.48493570751550,
        1.46598909439116,  2.48432584163895,  3.58182348355193,
        4.85946282833231};
wq = { 0.00000431065265, 0.00075807095698, 0.01911158107317,
       0.13548370704150, 0.34464234526294, 0.34464234526294,
       0.13548370704150, 0.01911158107317, 0.00075807095698,
       0.00000431065265};

/* initialize to zero */
 grp0a  =  J(4,1,0);    grp0b = J(4,1,0);    grp0c = J(4,1,0);
 grp1a  =  J(4,1,0);    grp1b = J(4,1,0);    grp1c = J(4,1,0);
```

```
DO q = 1 to nq;
    za0 = u0*gama + inta + sda*bq[q];
    zb0 = u0*gamb + intb + sdb*bq[q];
    za1 = u1*gama + inta + sda*bq[q];
    zb1 = u1*gamb + intb + sdb*bq[q];

    grp0a = grp0a + 1 / (1+(EXP(za0)+EXP(zb0)))*wq[q];
    grp0b = grp0b + EXP(za0) / (1+(EXP(za0)+EXP(zb0)))*wq[q];
    grp0c = grp0c + EXP(zb0) / (1+(EXP(za0)+EXP(zb0)))*wq[q];

    grp1a = grp1a + 1 / (1+(EXP(za1)+EXP(zb1)))*wq[q];
    grp1b = grp1b + EXP(za1) / (1+(EXP(za1)+EXP(zb1)))*wq[q];
    grp1c = grp1c + EXP(zb1) / (1+(EXP(za1)+EXP(zb1)))*wq[q];

END;

print 'Quadrature method - 10 points';
print 'marginal prob for group 0 - category 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - category 2' grp0b [FORMAT=8.4];
print 'marginal prob for group 0 - category 3' grp0c [FORMAT=8.4];
print 'marginal prob for group 1 - category 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - category 2' grp1b [FORMAT=8.4];
print 'marginal prob for group 1 - category 3' grp1c [FORMAT=8.4];
```