

Mixed-Effects Models for Ordinal Outcomes

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This work was supported by National Institute of Mental Health Contract N44MH32056.

Why analyze as ordinal?

- Efficiency: Armstrong & Sloan (1989, Amer Jrn of Epid) report efficiency losses between 89% to 99% comparing an ordinal to continuous outcome, depending on the number of categories and distribution within the ordinal categories.
- Bias: continuous model can yield correlated residuals and regressors when applied to ordinal outcomes, because the continuous model does not take into account the ceiling and floor effects of the ordinal outcome. This can result in biased estimates of regression coefficients and is most critical when the ordinal variables is highly skewed.
- Logic: continuous model can yield predicted values outside of the range of the ordinal variable.

Proportional Odds Model - McCullagh (1980)

$$\log \left[\frac{P(\mathbf{y} \leq c)}{1 - P(\mathbf{y} \leq c)} \right] = \gamma_c - \mathbf{x}'\boldsymbol{\beta}$$

$c = 1, \dots, C - 1$ for the C categories of the ordinal outcome

\mathbf{x} = vector of explanatory variables (plus the intercept)

γ_c = threshold parameters; reflect cumulative odds when $\mathbf{x} = 0$
(for identification: $\gamma_1 = 0$ or $\beta_0 = 0$)

- positive association between explanatory variable x and ordinal outcome variable y is reflected by β
- the effect of x is assumed to be the same for each cumulative odds ratio

- odds that the response is less than or equal to c (for any fixed c) is divided by e^β for every unit change in x :

$$\left[\frac{P(\mathbf{y} \leq c)}{1 - P(\mathbf{y} \leq c)} \right] = \exp(\gamma_c - x\beta) = e^{\gamma_c} / (e^\beta)^x$$

- the odds that the response is greater than or equal to c (for fixed c) is multiplied by e^β for every unit change in x :

$$\left[\frac{1 - P(\mathbf{y} \leq c)}{P(\mathbf{y} \leq c)} \right] = e^{-\gamma_c} \times (e^\beta)^x$$

Ordinal Model for Dichotomous Response (y coded 0 or 1)

$$\log \left[\frac{P(y = 0)}{1 - P(y = 0)} \right] = 0 - \mathbf{x}'\boldsymbol{\beta}$$

$$\frac{P(y = 0)}{1 - P(y = 0)} = \exp(0 - \mathbf{x}'\boldsymbol{\beta})$$

$$\frac{1 - P(y = 0)}{P(y = 0)} = [\exp(0 - \mathbf{x}'\boldsymbol{\beta})]^{-1}$$

$$\frac{1 - P(y = 0)}{P(y = 0)} = \exp(\mathbf{x}'\boldsymbol{\beta})$$

$$\log \left[\frac{P(y = 1)}{1 - P(y = 1)} \right] = \mathbf{x}'\boldsymbol{\beta}$$

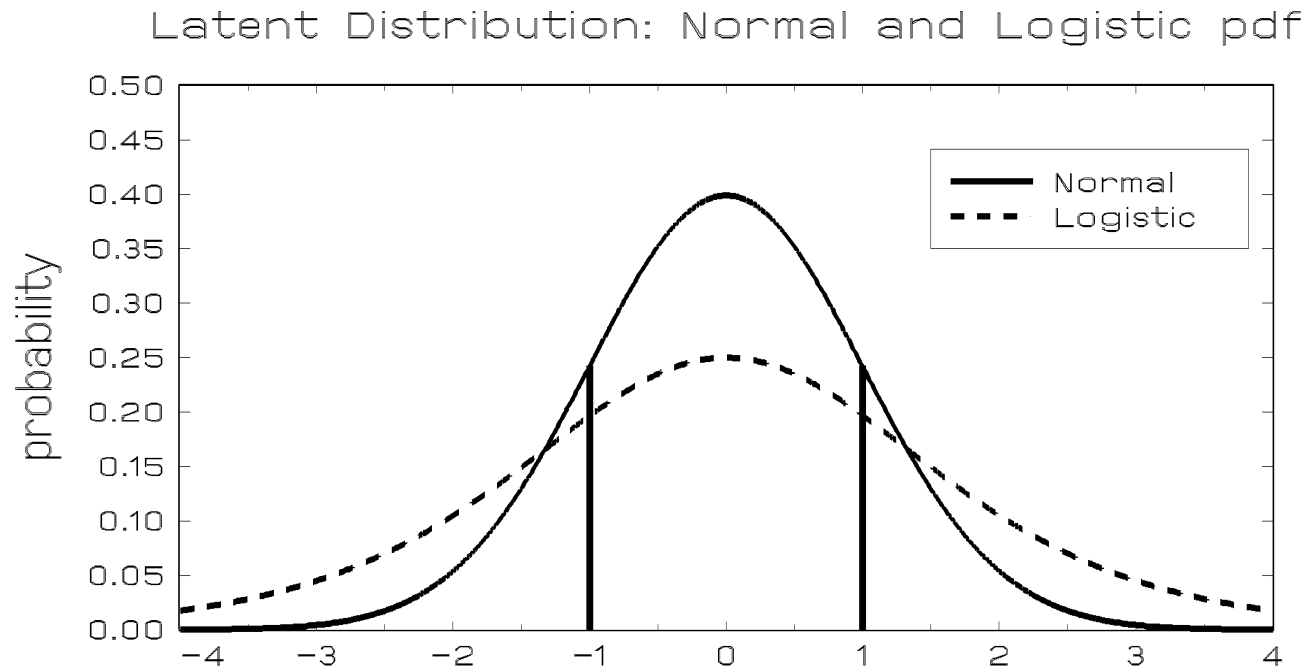
- same as it ever was!

Ordinal Response and Threshold Concept

Continuous y_{ij} - an unobservable latent variable - related to ordinal response y_{ij} via “threshold concept”

- series of threshold values $\gamma_1, \gamma_2, \dots, \gamma_{C-1}$
- $C =$ number of ordered categories, and $\gamma_0 = -\infty$ and $\gamma_C = \infty$

Response occurs in category c , $y_{ij} = c$ if $\gamma_{c-1} < y_{ij} < \gamma_c$




The Threshold Concept in Practice

“How was your day?” (what is your satisfaction level today?)

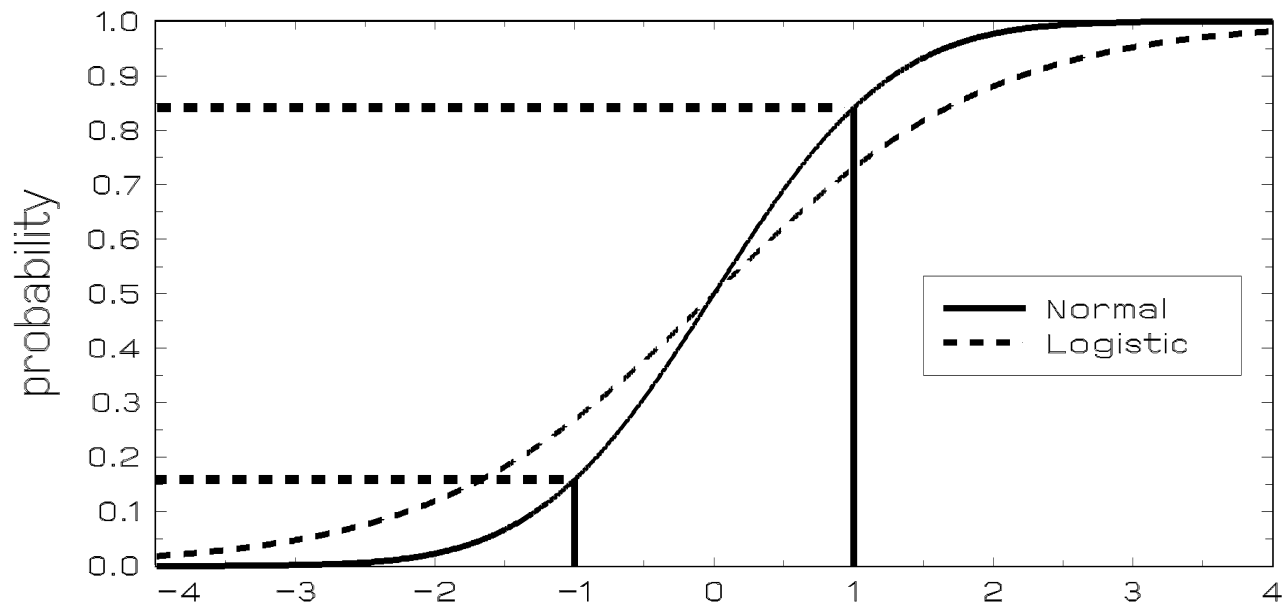
- Satisfaction may be continuous, but we sometimes emit an ordinal response:

 **Great Day!**

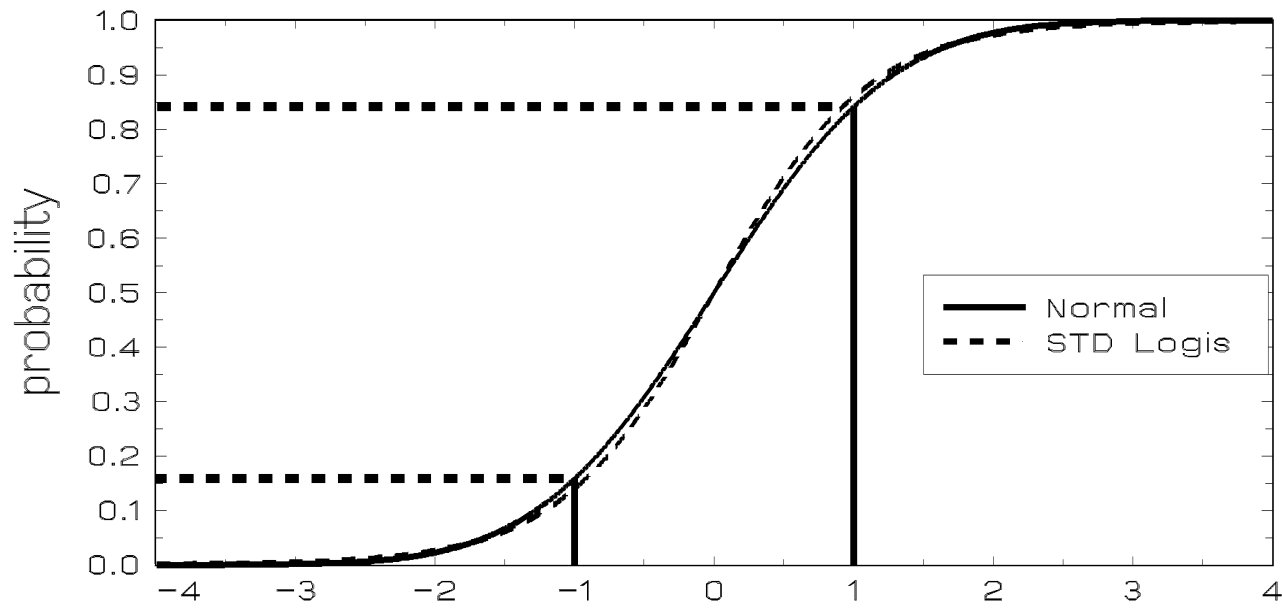
 **a day ...**

 ***?!**!? day**

Latent Distribution: Normal and Logistic cdf



Latent Dist: Normal & STD Logistic cdf



Ordinal model and logistic cdf

The model for cumulative logits (proportional odds model)

$$\log \left[\frac{P(\mathbf{y}_i \leq c)}{1 - P(\mathbf{y}_i \leq c)} \right] = \gamma_c - \mathbf{x}'_i \boldsymbol{\beta}$$

can be re-written as:

$$P(\mathbf{y}_i \leq c) = \Psi(\gamma_c - \mathbf{x}'_i \boldsymbol{\beta})$$

where for identification: $\gamma_1 = 0$ (or $\beta_0 = 0$) and the cumulative std logistic dist fn (cdf) is

$$\Psi(\gamma_c - \mathbf{x}'_i \boldsymbol{\beta}) = \frac{1}{1 + \exp[-(\gamma_c - \mathbf{x}'_i \boldsymbol{\beta})]}$$

The probability of a response in a given category is then:

$$P(\mathbf{y}_i = c) = \Psi(\gamma_c - \mathbf{x}'_i \boldsymbol{\beta}) - \Psi(\gamma_{c-1} - \mathbf{x}'_i \boldsymbol{\beta})$$

Mixed-effects ordinal logistic regression model: generalization of proportional odds model

Model for cumulative logits

$$\log \left[\frac{P(y_{ij} \leq c)}{1 - P(y_{ij} \leq c)} \right] = \gamma_c - [\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{v}_i]$$

\Rightarrow random-effect generalization of proportional odds model (with $\mathbf{v}_i \sim \mathcal{NID}(\mathbf{0}, \boldsymbol{\Sigma}_v)$)

Note, as in proportional odds model:

- origin of y : $\gamma_1 = 0$
- unit of measurement: $\sigma = \pi/\sqrt{3}$

Category probabilities

$$P(\mathbf{y}_{ij} \leq c) = \Psi(\gamma_c - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\boldsymbol{\nu}_i))$$

$$P(\mathbf{y}_{ij} = c) = \Psi(\gamma_c - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\boldsymbol{\nu}_i)) - \Psi(\gamma_{c-1} - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\boldsymbol{\nu}_i))$$

where the cumulative std logistic dist fn (cdf) is

$$\Psi(\gamma_c - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\boldsymbol{\nu}_i)) = \frac{1}{1 + \exp[-(\gamma_c - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\boldsymbol{\nu}_i))]}$$

Cumulative Link Models

$$G^{-1} [P(\mathbf{y}_{ij} \leq c)] = \gamma_c - [\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\boldsymbol{\nu}_i]$$

or, equivalently

$$P(\mathbf{y}_{ij} \leq c) = G [\gamma_c - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\boldsymbol{\nu}_i)]$$

where

- $G^{-1}(P) = \log[P/(1 - P)]$ gives cumulative logit models (proportional odds models)
- $G^{-1}(P) = \Phi^{-1}(P)$ gives cumulative probit models
- $G^{-1}(P) = \log[-\log(1 - P)]$ (complementary log-log link) gives proportional hazards models

Multilevel Representation

Within level-2 unit model (*level-1 model*)

observed response

$$\log \left[\frac{\Pr(y_{ij} \leq c)}{1 - \Pr(y_{ij} \leq c)} \right] = \gamma_c - \left[\mathbf{z}'_{(1)ij} \mathbf{b}_i + \mathbf{x}'_{(1)ij} \boldsymbol{\beta}_{(1)} \right]$$

or

latent response

$$y_{ij} = \mathbf{z}'_{(1)ij} \mathbf{b}_i + \mathbf{x}'_{(1)ij} \boldsymbol{\beta}_{(1)} + \varepsilon_{ij}$$

Between level-2 unit model

$$\mathbf{b}_i = \boldsymbol{\beta}_{0(2)} + \mathbf{x}'_{(2)i} \boldsymbol{\beta}_{(2)} + \mathbf{v}_i$$

- $\mathbf{x}_{(1)ij}$ and $\boldsymbol{\beta}_{(1)}$ are level-1 covariates and effects
- $\mathbf{x}_{(2)i}$ and $\boldsymbol{\beta}_{(2)}$ are level-2 covariates and effects
- $\mathbf{z}_{(1)ij}$ are level-1 variables that vary at level-2
- $\varepsilon_{ij} \sim \mathcal{LID}(0, \sigma^2)$ for logistic

The level-2 effects \mathbf{b}_i are a function of

- an overall mean $\boldsymbol{\beta}_0(2)$
- level-2 covariates $\boldsymbol{\beta}_{(2)}$
- a unique random component $\mathbf{v}_i \sim \mathcal{NID}(\mathbf{0}, \boldsymbol{\Sigma}_v)$
(level-2 residuals)

Effects of a School-based Intervention

The Television School and Family Smoking Prevention and Cessation Project (Flay, *et al.*, 1988); a subsample:

- *sample* - 1600 7th-graders - 135 classes - 28 schools
 - 1 to 13 classes per school, 2 to 28 students per class
- *outcome* - knowledge of the effects of tobacco use
- *timing* - students tested at pre and post-intervention
- *design* - schools exposed to
 - a social-resistance classroom curriculum (CC)
 - a media (television) intervention (TV)
 - CC combined with TV
 - a no-treatment control group

Main question of interest:

- Influence of the intervention on the tobacco health knowledge scores (THKS) ?

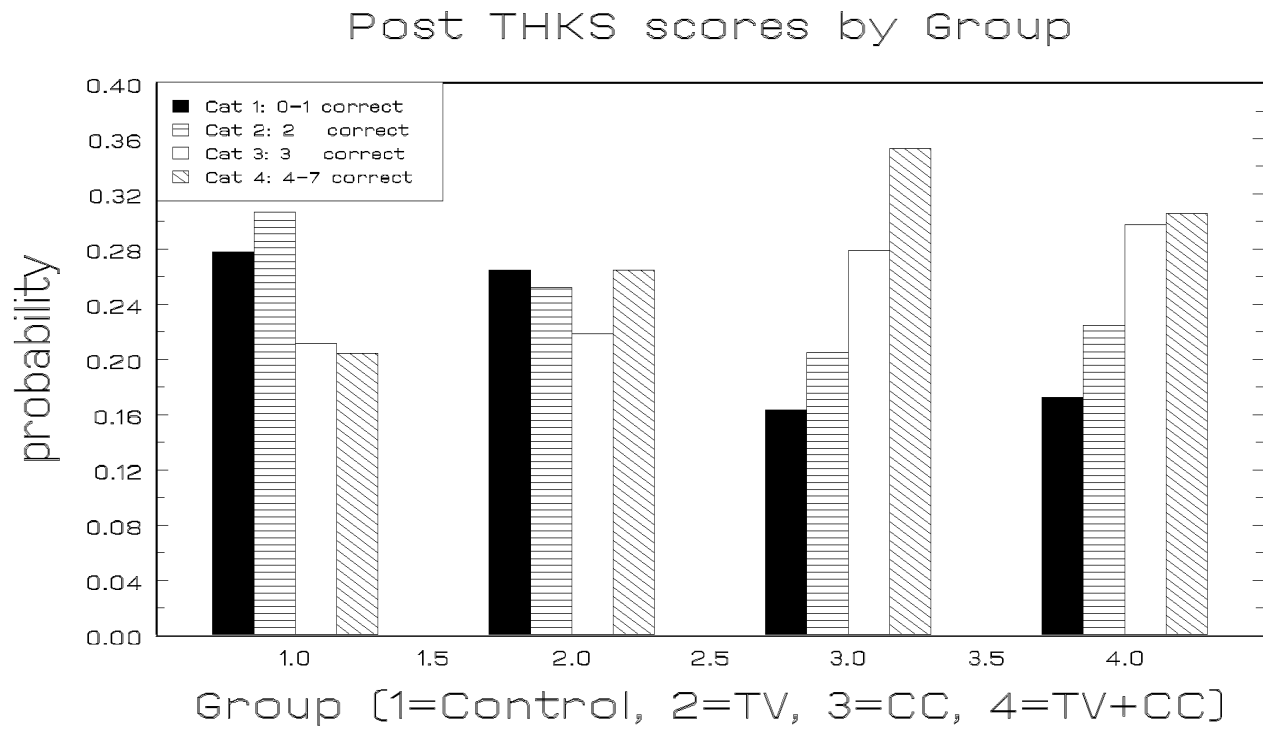
Challenges in the analysis:

- outcome variable (THKS) is number correct of 7 items
- controlling for intra-school and intra-class variability
- potential explanatory variables are at different levels

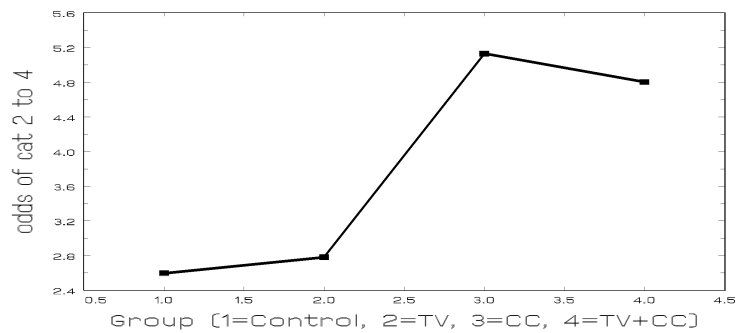
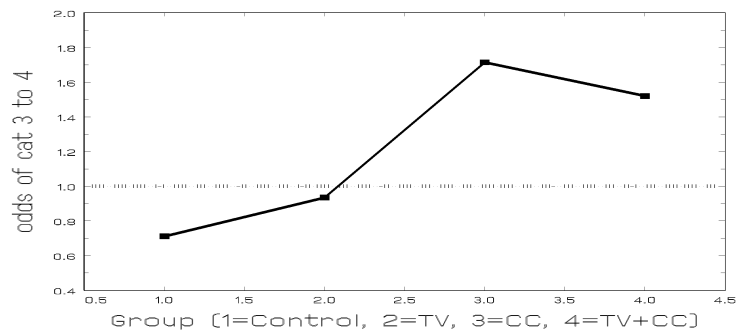
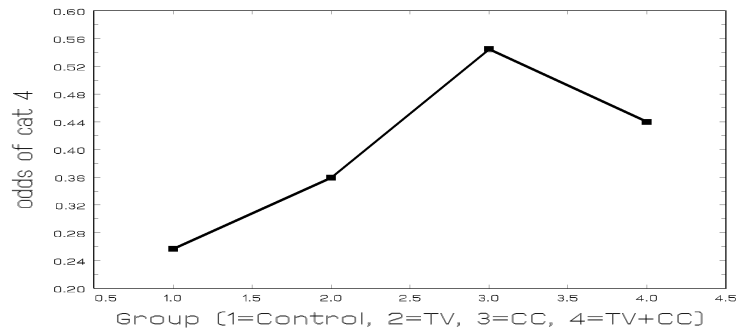
Tobacco and Health Knowledge Scale
 Post-Intervention Scores - Frequencies (percentages)

<i>subgroup</i>		<i>THKS score</i>				
CC	TV	0-1	2	3	4-7	<i>total</i>
no	no	117 (27.8)	129 (30.6)	89 (21.1)	86 (20.4)	421
no	yes	110 (26.4)	105 (25.2)	91 (21.9)	110 (26.4)	416
yes	no	62 (16.3)	78 (20.5)	106 (27.9)	134 (35.3)	380
yes	yes	66 (17.2)	86 (22.5)	114 (29.8)	117 (30.5)	383
<i>total</i>		355 (22.2)	398 (24.9)	400 (25.0)	447 (27.9)	1600

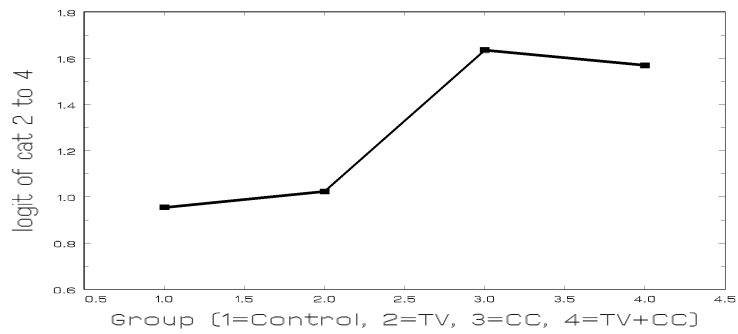
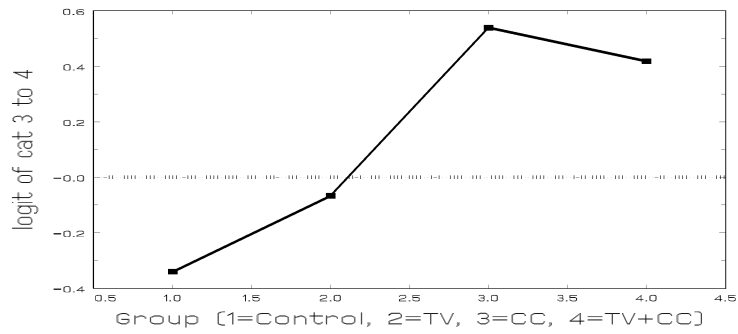
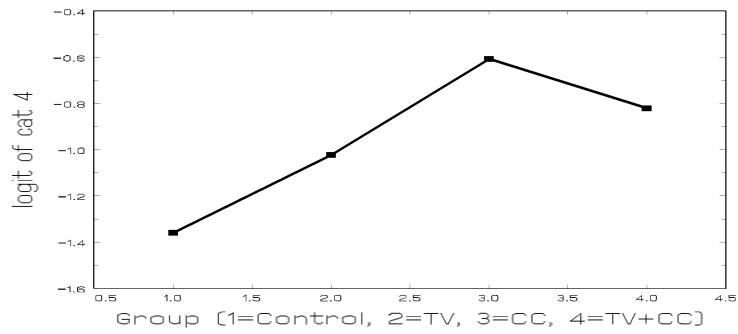
Observed Proportions by Group



Observed Odds by Group



Observed Logits by Group



Within-Subjects / Between-Subjects components

Within-clusters model - level 1 ($j = 1, \dots, n_i$ subjects)

$$\text{logit}_{ijc} = \gamma_c - [b_{0i} + b_{1i}PRETHKS_{ij}]$$

Between-clusters model - level 2 ($i = 1, \dots, N$ clusters)

$$b_{0i} = \beta_0 + \beta_2CC_i + \beta_3TV_i + \beta_4(CC_i \times TV_i) + v_{0i}$$

$$b_{1i} = \beta_1$$

$$v_{0i} \sim \mathcal{NID}(0, \sigma_v^2)$$

- $0 - \beta_0$ = 1st logit (1 vs 2-4) for CC=no TV=no subgroup
(PRETHKS adjusted)
- $\gamma_2 - \beta_0$ = 2nd logit (1-2 vs 3-4) for CC=no TV=no subgroup
(PRETHKS adjusted)
- $\gamma_3 - \beta_0$ = 3rd logit (1-3 vs 4) for CC=no TV=no subgroup
(PRETHKS adjusted)
- β_1 = effect of PRETHKS on POSTTHKS
- β_2 = (PRETHKS adjusted) logit diff. between CC=yes vs
CC=no (for TV=no)
- β_3 = (PRETHKS adjusted) logit diff. between TV=yes vs
TV=no (for CC=no)
- β_4 = (PRETHKS adjusted) difference in logit attributable
to interaction
- ν_{0i} = random cluster deviation

THKS Post Intervention (ordinal) Scores - LR Estimates (standard errors)

	<i>Ordinary LR</i>		<i>Mixed LR models</i>	
intercept	.040		.076	
	(.123)		(.154)	
threshold ₂	1.225 ***		1.273 ***	
	(.056)		(.063)	
threshold ₃	2.385 ***		2.479 ***	
	(.072)		(.080)	
PRETHKS	.422 ***		.415 ***	
	(.037)		(.041)	
CC	.863 ***		.861 ***	
	(.132)		(.187)	
TV	.253 **		.206	
	(.125)		(.168)	
CC × TV	-.367 **		-.301	
	(.183)		(.252)	
cluster sd		class = .434 [<i>r</i> = .054]	school = .271 [<i>r</i> = .022]	
		(.076)	(.091)	
-2 log L	4250.21	4230.77	4239.49	
*** <i>p</i> < .01 ** <i>p</i> < .05 * <i>p</i> < .10 ; (Wald-tests not done for sds)				

Model fit of proportions: Random schools model

CC TV logistic $\Psi(z) = \frac{1}{1+\exp(-z)}$ est.

Probability of Category 1 response

0 0	$\Psi((0 - (.089 + 2.152 \times .403))/\sqrt{\hat{d}})$.280
0 1	$\Psi((0 - (.089 + .275 + 2.087 \times .403))/\sqrt{\hat{d}})$.233
1 0	$\Psi((0 - (.089 + .923 + 2.050 \times .403))/\sqrt{\hat{d}})$.140
1 1	$\Psi((0 - (.089 + .275 + .923 - .466 + 1.979 \times .403))/\sqrt{\hat{d}})$.168

Probability of Category 1 or 2 response

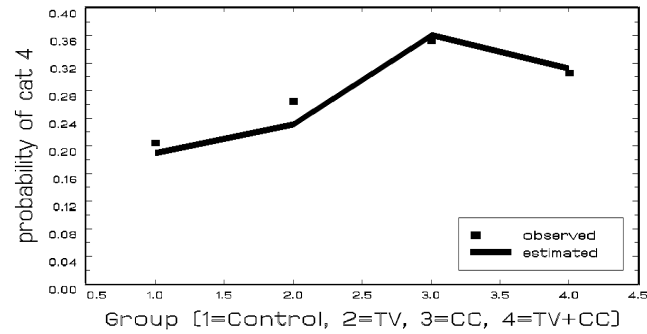
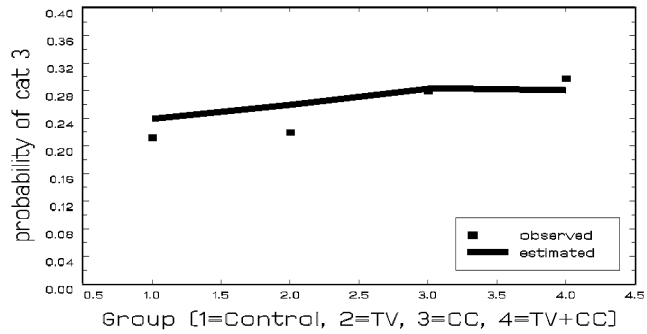
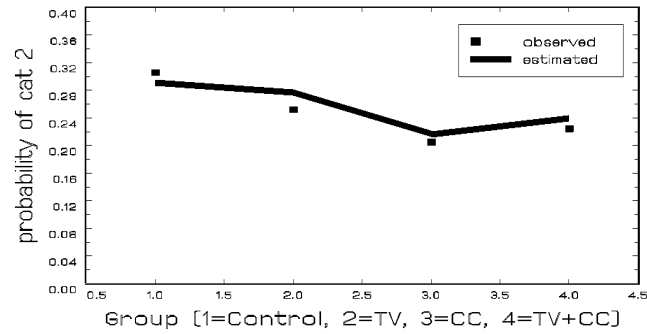
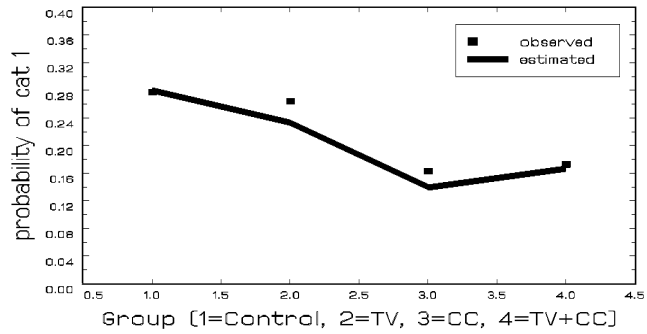
0 0	$\Psi((1.242 - (.089 + 2.152 \times .403))/\sqrt{\hat{d}})$.570
0 1	$\Psi((1.242 - (.089 + .275 + 2.087 \times .403))/\sqrt{\hat{d}})$.509
1 0	$\Psi((1.241 - (.089 + .923 + 2.050 \times .403))/\sqrt{\hat{d}})$.357
1 1	$\Psi((1.242 - (.089 + .275 + .923 - .466 + 1.979 \times .403))/\sqrt{\hat{d}})$.408

Probability of Category 1, 2, or 3 response

0 0	$\Psi((2.420 - (.089 + 2.152 \times .403))/\sqrt{\hat{d}})$.810
0 1	$\Psi((2.420 - (.089 + .275 + 2.087 \times .403))/\sqrt{\hat{d}})$.769
1 0	$\Psi((2.420 - (.089 + .923 + 2.050 \times .403))/\sqrt{\hat{d}})$.640
1 1	$\Psi((2.420 - (.089 + .275 + .923 - .466 + 1.979 \times .403))/\sqrt{\hat{d}})$.689

$d = \text{design effect} = (\sigma_v^2 + \sigma^2)/\sigma^2 \quad \hat{d} = (.271^2 + \pi^2/3)/(\pi^2/3)$

Model Fit of Observed Proportions: random schools model



Treatment-Related Change Across Time

Data from the NIMH Schizophrenia collaborative study on treatment related changes in overall severity. IMPS item 79, *Severity of Illness*, was scored as:

- 1 = normal or borderline mentally ill
- 2 = mildly or moderately ill
- 3 = markedly ill
- 4 = severely or among the most extremely ill

The experimental design and corresponding sample sizes:

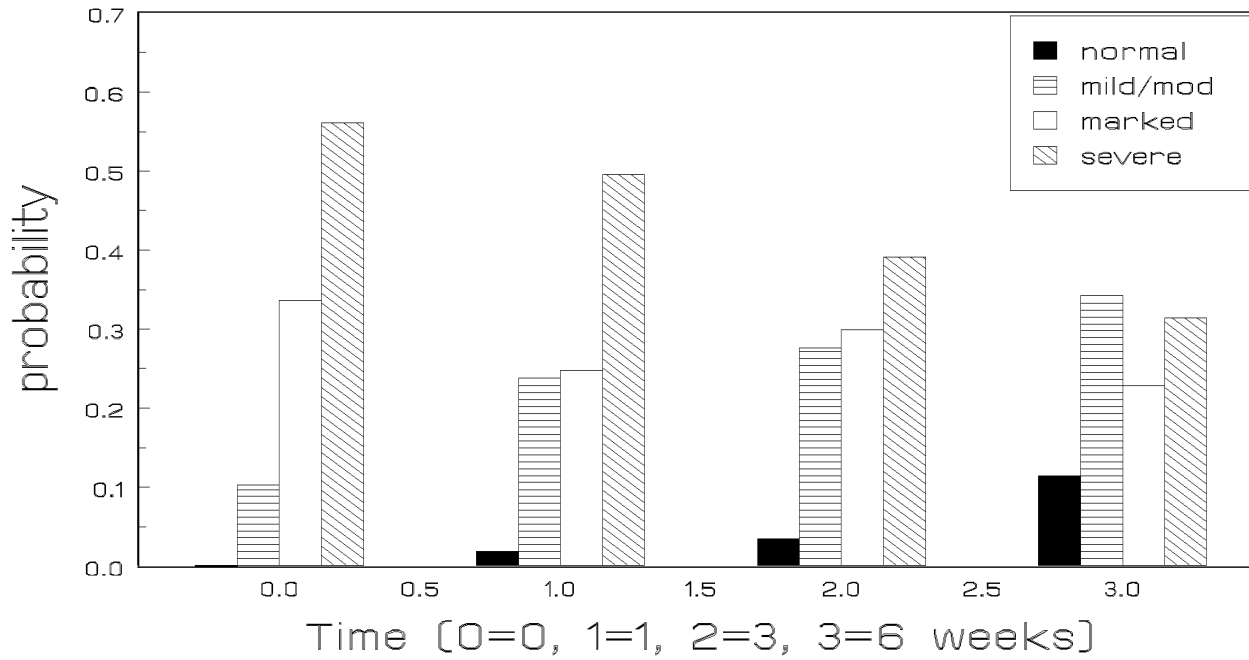
Group	Sample size at Week							<i>completers</i>
	0	1	2	3	4	5	6	
PLC (n=108)	107	105	5	87	2	2	70	65%
DRUG (n=329)	327	321	9	287	9	7	265	81%

Drug = Chlorpromazine, Fluphenazine, or Thioridazine

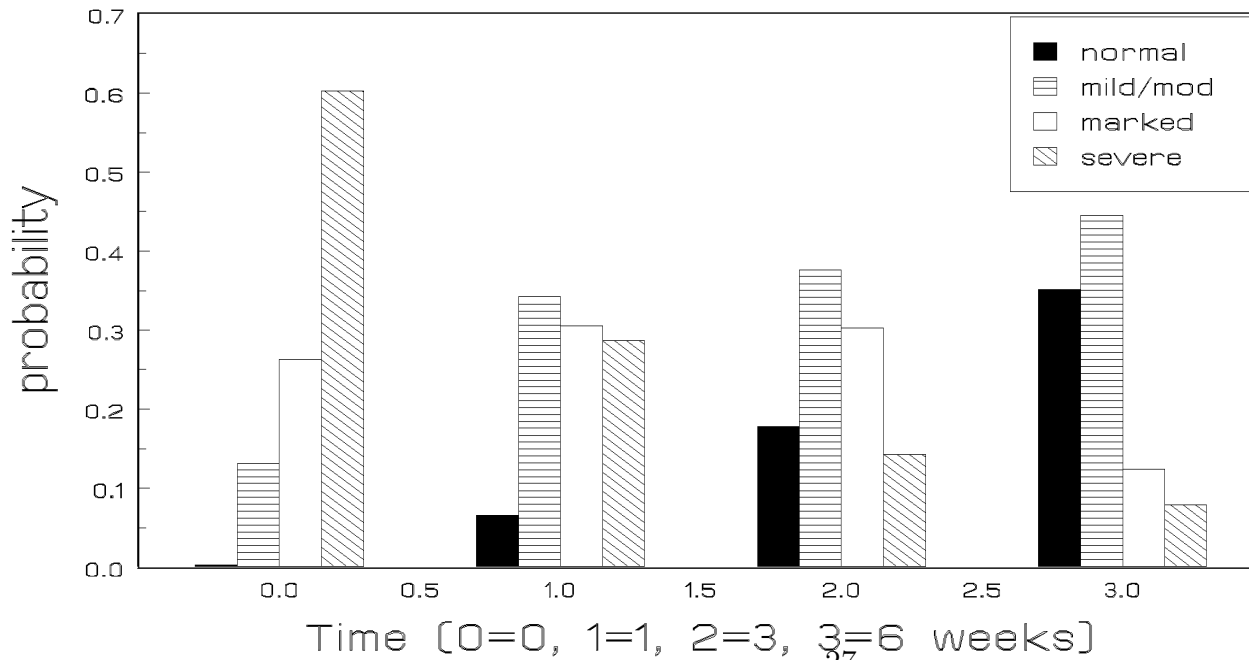
Main question of interest:

- Was there differential improvement for the drug groups relative to the control group?

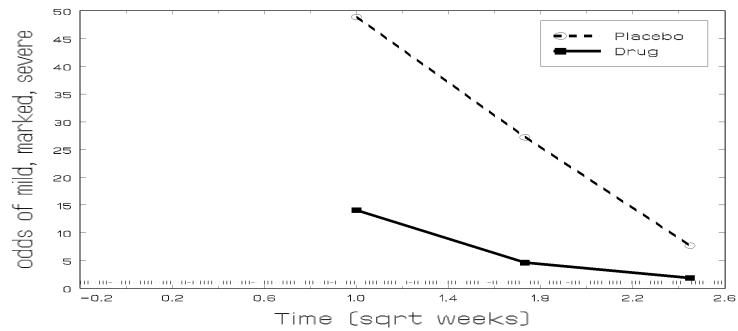
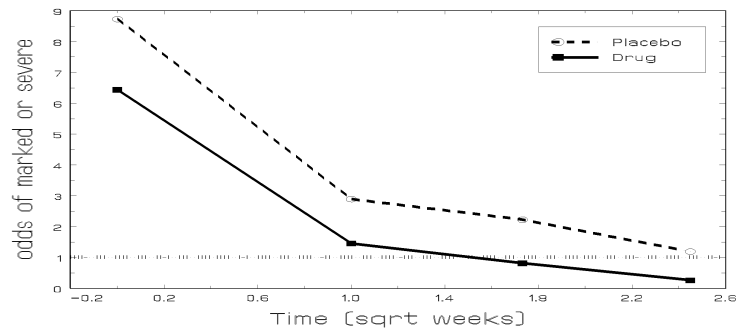
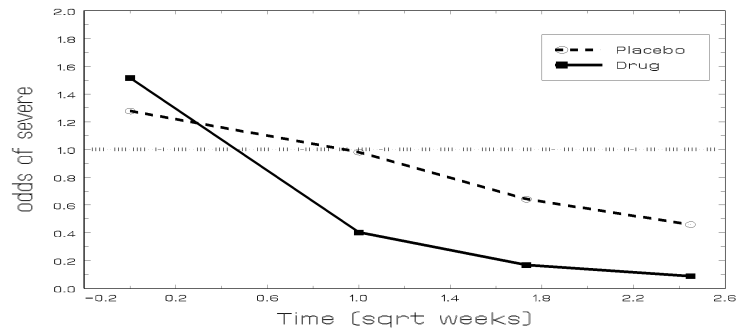
IMPS 79 Severity by Time: Control



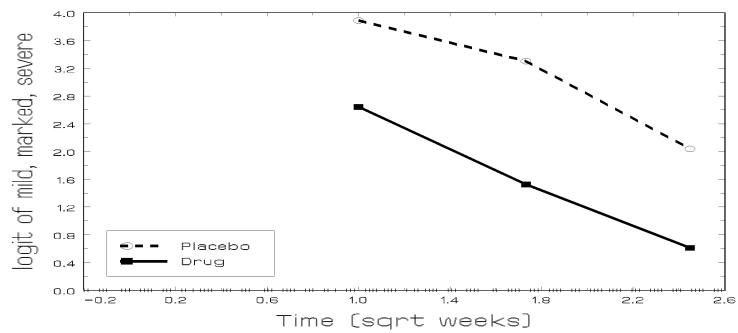
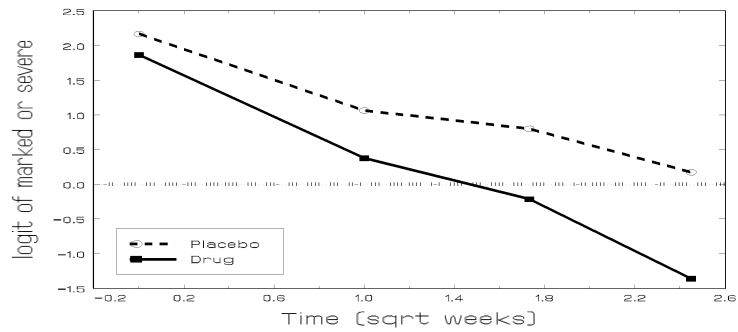
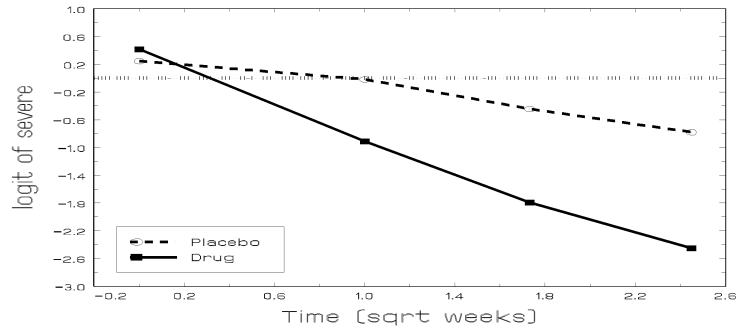
IMPS 79 Severity by Time: Drug



Observed Odds across Time by Condition



Observed Logits across Time by Condition



Within-Subjects / Between-Subjects components

Within-subjects model - level 1 ($j = 1, \dots, n_i$ obs)

$$\text{logit}_{ijc} = \gamma_c - [b_{0i} + b_{1i}\sqrt{\text{Week}_j}]$$

Between-subjects model - level 2 ($i = 1, \dots, N$ subjects)

$$b_{0i} = \beta_0 + \beta_2 \text{Grp}_i + v_{0i}$$

$$b_{1i} = \beta_1 + \beta_3 \text{Grp}_i$$

$$v_{0i} \sim \mathcal{NID}(0, \sigma_v^2)$$

- $0 - \beta_0$ = week 0 IMPS79 1st logit (1 vs 2-4)
 $\gamma_2 - \beta_0$ = week 0 IMPS79 2nd logit (1-2 vs 3-4)
 $\gamma_3 - \beta_0$ = week 0 IMPS79 3rd logit (1-3 vs 4)
- β_1 = IMPS79 (sqrt) weekly logit change for PLC patients
 ($Grp = 0$)
- β_2 = difference in week 0 IMPS79 logit for DRUG patients
 ($Grp = 1$)
- β_3 = difference in IMPS79 (sqrt) weekly logit change for
 DRUG patients ($Grp = 1$)
- u_{0i} = individual deviation from group trend

NIMH Schiz Study: Severity of Illness (N = 437)
 Ordinal LR Estimates (se) - *random intercept model*

	ML estimates	se	z	$p <$
intercept	5.858	0.343	17.08	.001
threshold ₂	3.033	0.132	22.91	.001
threshold ₃	5.152	0.179	28.74	.001
Drug (0 = plc; 1 = drug)	-0.055	0.311	-0.18	.86
Time (sqrt week)	-0.766	0.120	-6.39	.001
Drug by Time	-1.206	0.133	-9.06	.001
Intercept sd	1.944	0.128		
<i>Intra-person correlation</i> = $1.944^2 / (1.944^2 + \pi^2/3) = .53$				
$-2 \log L = 3402.72$				

Within-Subjects / Between-Subjects components

Within-subjects model - level 1 ($j = 1, \dots, n_i$ obs)

$$\text{logit}_{ijc} = \gamma_c - [b_{0i} + b_{1i}\sqrt{\text{Week}_j}]$$

Between-subjects model - level 2 ($i = 1, \dots, N$ subjects)

$$b_{0i} = \beta_0 + \beta_2 \text{Grp}_i + v_{0i}$$

$$b_{1i} = \beta_1 + \beta_3 \text{Grp}_i + v_{1i}$$

$$\mathbf{v}_i \sim \mathcal{NID}(\mathbf{0}, \Sigma_v = \mathbf{T}\mathbf{T}')$$

- $0 - \beta_0$ = week 0 IMPS79 1st logit (1 vs 2-4)
 $\gamma_2 - \beta_0$ = week 0 IMPS79 2nd logit (1-2 vs 3-4)
 $\gamma_3 - \beta_0$ = week 0 IMPS79 3rd logit (1-3 vs 4)
- β_1 = IMPS79 (sqrt) weekly logit change for PLC patients
 ($Grp = 0$)
- β_2 = difference in week 0 IMPS79 logit for DRUG patients
 ($Grp = 1$)
- β_3 = difference in IMPS79 (sqrt) weekly logit change for
 DRUG patients ($Grp = 1$)
- v_{0i} = individual deviation from group intercept
 v_{1i} = individual deviation from group (sqrt) weekly change

NIMH Schiz Study: Severity of Illness ($N = 437$)

Ordinal LR Estimates (se) - *random intercept and trend model*

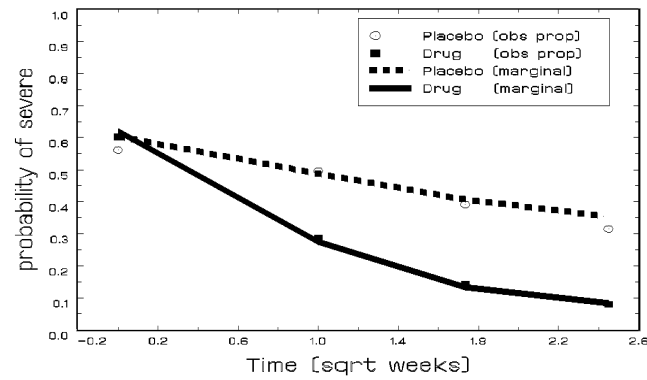
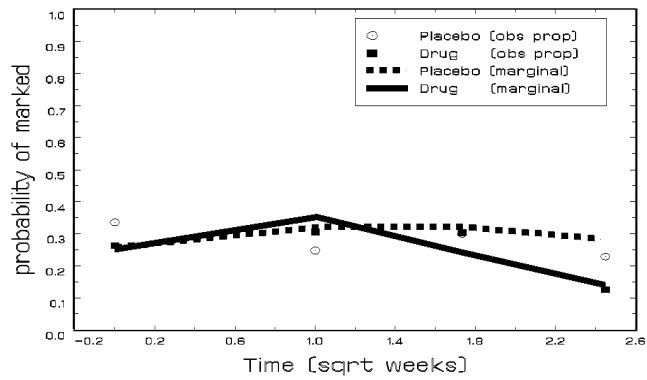
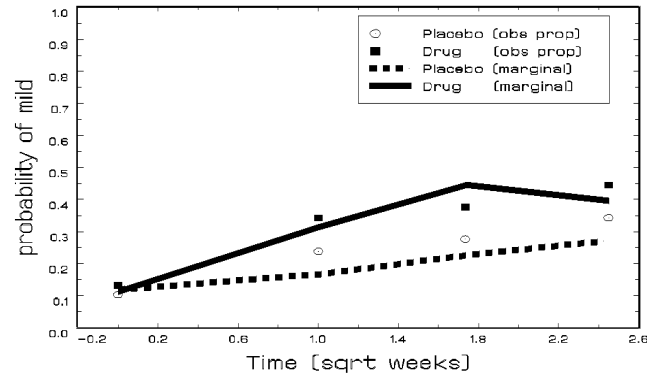
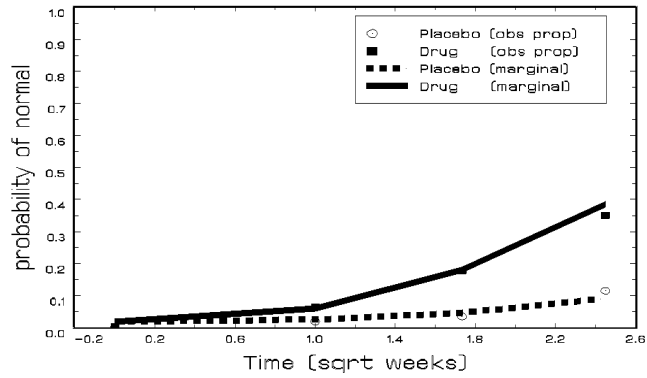
	ML estimates	se	z	$p <$
intercept	7.309	0.484	15.10	.001
threshold ₂	3.912	0.214	18.29	.001
threshold ₃	6.528	0.291	22.43	.001
Drug 0=plc; 1 = drug)	0.111	0.402	0.28	.78
Time (sqrt week)	-0.875	0.236	-3.71	.001
Drug by Time	-1.724	0.270	-6.40	.001
<i>Cholesky</i>				
Intercept sd	2.669	0.256	$(\hat{\sigma}_{v_0}^2 = 7.13)$	
Int-Time covar	-0.588	0.159	$(r_{v_0v_1} = -.41)$	
Time sd	1.308	0.126	$(\hat{\sigma}_{v_1}^2 = 2.06)$	

$-2 \log L = 3324.82, \chi_2^2 = 77.90, p < .001$

Model Fit of Observed Proportions

condition	category	model	week 0	week 1	week 3	week 6
Placebo	normal	<i>observed</i>	.00	.02	.03	.11
		random int	.02	.03	.05	.07
		random int & trend	.02	.03	.05	.09
	mild/mod	<i>observed</i>	.10	.24	.28	.34
		random int	.11	.17	.22	.28
		random int & trend	.12	.17	.23	.27
	marked	<i>observed</i>	.34	.25	.30	.23
		random int	.25	.30	.33	.33
		random int & trend	.26	.32	.32	.28
	severe	<i>observed</i>	.56	.50	.39	.31
		random int	.61	.49	.40	.32
		random int & trend	.60	.49	.41	.35
Drug	normal	<i>observed</i>	.00	.07	.18	.35
		random int	.02	.07	.17	.35
		random int & trend	.02	.06	.18	.39
	mild/mod	<i>observed</i>	.13	.34	.38	.45
		random int	.12	.30	.43	.45
		random int & trend	.11	.31	.45	.40
	marked	<i>observed</i>	.26	.31	.30	.12
		random int	.26	.33	.26	.14
		random int & trend	.25	.35	.24	.14
	severe	<i>observed</i>	.60	.29	.14	.08
		random int	.61	.30	.14	.06
		random int & trend	.62	.27	.13	.08

Model Fit of Observed Proportions



```

TITLE1 'NIMH Schizophrenia Data - Estimated Marginal Probabilities';
PROC IML;
/* Results from MIXOR analysis: random intercept model */;
x0 = { 0 0.00000 0,
      0 1.00000 0,
      0 1.73205 0,
      0 2.44949 0};
x1 = { 1 0.00000 0.00000,
      1 1.00000 1.00000,
      1 1.73205 1.73205,
      1 2.44949 2.44949};
int   = {5.858};
sd    = {1.944};
beta  = {-.055, -.766, -1.206};
thresh = {3.033, 5.152};
/* Approximate Marginalization Method */;
pi    = 3.141592654;
nt    = 4;
ivec  = J(nt,1,1);
zvec  = J(nt,1,1);
evec  = (15/16)**2 * (pi**2)/3 * ivec;
/* nt by nt matrix with evec on the diagonal and zeros elsewhere */;
emat = diag(evec);

/* variance-covariance matrix of underlying latent variable */;
vary = zvec * sd * T(sd) * T(zvec) + emat;

sdy = sqrt(vecdiag(vary) / vecdiag(emat));

```

```

za0 = (0 - (int + x0*beta)) / sdy ;
zb0 = (thresh[1] - (int + x0*beta)) / sdy;
zc0 = (thresh[2] - (int + x0*beta)) / sdy;
za1 = (0 - (int + x1*beta)) / sdy;
zb1 = (thresh[1] - (int + x1*beta)) / sdy;
zc1 = (thresh[2] - (int + x1*beta)) / sdy;

grp0a = 1 / ( 1 + EXP(0 - za0));
grp0b = 1 / ( 1 + EXP(0 - zb0));
grp0c = 1 / ( 1 + EXP(0 - zc0));
grp1a = 1 / ( 1 + EXP(0 - za1));
grp1b = 1 / ( 1 + EXP(0 - zb1));
grp1c = 1 / ( 1 + EXP(0 - zc1));

print 'Random intercept model';
print 'Approximate Marginalization Method';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (grp0c-grp0b) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 4' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (grp1c-grp1b) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 4' (1-grp1c) [FORMAT=8.4];

```

```

/* Random Intercept and Trend Model */;

int    = {7.309};
chol   = {2.669 0,
          -.588 1.308};
beta   = { .111, -.875, -1.724};
thresh = {3.912, 6.528};

/* Approximate Marginalization Method */;

pi     = 3.141592654;
nt     = 4;
ivec   = J(nt,1,1);
zmat   = {1 0.00000,
          1 1.00000,
          1 1.73205,
          1 2.44949};
evec   = (15/16)**2 * (pi**2)/3 * ivec;

/* nt by nt matrix with evec on the diagonal and zeros elsewhere */;
emat = diag(evec);

/* variance-covariance matrix of underlying latent variable */;
vary = zmat * chol * T(chol) * T(zmat) + emat;

sdy = sqrt(vecdiag(vary) / vecdiag(emat));

```

```

za0 = (0 - (int + x0*beta)) / sdy ;
zb0 = (thresh[1] - (int + x0*beta)) / sdy;
zc0 = (thresh[2] - (int + x0*beta)) / sdy;
za1 = (0 - (int + x1*beta)) / sdy;
zb1 = (thresh[1] - (int + x1*beta)) / sdy;
zc1 = (thresh[2] - (int + x1*beta)) / sdy;

grp0a = 1 / ( 1 + EXP(0 - za0));
grp0b = 1 / ( 1 + EXP(0 - zb0));
grp0c = 1 / ( 1 + EXP(0 - zc0));
grp1a = 1 / ( 1 + EXP(0 - za1));
grp1b = 1 / ( 1 + EXP(0 - zb1));
grp1c = 1 / ( 1 + EXP(0 - zc1));

print 'Random intercept and trend model';
print 'Approximate Marginalization Method';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (grp0c-grp0b) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 4' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (grp1c-grp1b) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 4' (1-grp1c) [FORMAT=8.4];

```