

# Factor Analysis by MINRES

To the memory of Harry Harman and Henry Kaiser

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The basic idea of *exploratory factor analysis* is the following. For a given set of observed response variables one wants to find a set of latent factors, fewer in number than the observed variables. These factors are supposed to account for the intercorrelations of the response variables in the sense that when the factors are partialled out from the observed variables, there should no longer remain any correlations between these. The term *exploratory* means that one does not know how many factors are needed nor does one know the factorial nature of the observed variables. Before 1970 the term *factor analysis* was used without the word *exploratory*; this term was added later to distinguish it from *confirmatory factor analysis* introduced around 1969 (see, e.g., Jöreskog, 1969)

The term MINRES (MINimum RESiduals) was first introduced by Harry Harman (1960) as a way of doing factor analysis by least squares. This was at a time when nobody cared about where the data came from or what distributions it had. It was only a matter of fitting a matrix of factor loadings to the correlation matrix.

A program EFAP for exploratory factor analysis was written by Jöreskog & Sörbom around 1977 based on algorithms for unweighted least squares (ULS), generalized least squares (GLS), and maximum likelihood (ML) by Jöreskog (1977). EFAP is obsolete and no longer available. However, exploratory factor analysis was reintroduced in LISREL 8.30, but only for ML, see Jöreskog, *et al.* (2000).

In this note I show that MINRES and ULS are equivalent and give the same solution except for an orthogonal transformation of the factor loadings. The MINRES presented here and introduced in LISREL 8.54 is based on the direct minimization of least squares rather the ULS minimization in Jöreskog (1977) which is based on eigenvalues and eigenvectors of the reduced correlation matrix.

Although ULS has been available for general covariance structures in LISREL since the introduction of LISREL V in 1985, see Sörbom (2001), it is seldom used. However, ULS is quite robust (see, textsle.g., Balderjahn, 1985) and deserves more attention. It does not require any distributional assumptions<sup>1</sup>. It can be used with small samples even when the number of variables is large and when the correlation matrix is not positive definite for other reasons (for example, this might be the case for a matrix of tetrachoric or polychoric correlations). It is particularly suited for exploratory factor analysis where only parameter estimates (and *not* standard error estimates and chi-square values) are of interest. For further explanation of exploratory factor analysis see Jöreskog (1979) and Jöreskog, *et al.* (2000) pp. 143–156.

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<sup>1</sup>In the LISREL implementation, a normality assumption is used only to obtain standard error estimates and *t*-values for the reference variable solution.

# 1 Equivalence of MINRES and ULS

Let  $\mathbf{R}$  be a given correlation matrix of order  $p \times p$  and let  $\mathbf{\Lambda}$  be a matrix of order  $p \times k$ , where  $p$  is the number of variables and  $k$  is the number of factors. The latter is usually much less than  $p$ . The problem is to estimate  $\mathbf{\Lambda}$  by minimizing the sum of squares

$$f(\mathbf{R}, \mathbf{\Lambda}) = \sum_{i=2}^p \sum_{j=1}^{i-1} (r_{ij} - \sum_{m=1}^k \lambda_{im} \lambda_{jm})^2, \quad (1)$$

with respect to  $\mathbf{\Lambda}$  subject to the constraints that

$$\sum_{m=1}^k \lambda_{im}^2 \leq 1, \quad i = 1, 2, \dots, p. \quad (2)$$

If equality holds for variable  $i$  in (2) we say that this variable is a Heywood case.

The ULS fit function is (see Jöreskog & Sörbom, 1999, p. 20)

$$F(\mathbf{S}, \mathbf{\Sigma}) = \frac{1}{2} \text{tr} [(\mathbf{S} - \mathbf{\Sigma})^2], \quad (3)$$

where  $\mathbf{S}$  is a covariance matrix and  $\mathbf{\Sigma} = \mathbf{\Lambda} \mathbf{\Lambda}' + \mathbf{\Theta}$  in this case, where  $\mathbf{\Theta}$  is a diagonal matrix of unique variances. Applying this to a correlation matrix gives

$$F(\mathbf{R}, \mathbf{\Lambda}, \mathbf{\Theta}) = \frac{1}{2} \text{tr} [(\mathbf{R} - \mathbf{\Lambda} \mathbf{\Lambda}' - \mathbf{\Theta})^2] \quad (4)$$

$$= \sum_{i=2}^p \sum_{j=1}^{i-1} (r_{ij} - \sum_{m=1}^k \lambda_{im} \lambda_{jm})^2 + \frac{1}{2} \sum_{i=1}^p (1 - \sum_{m=1}^k \lambda_{im}^2 - \theta_i)^2, \quad (5)$$

where  $\theta_i$  is the  $i$ th diagonal element of  $\mathbf{\Theta}$ . Disregarding the constraints (2) for the moment and minimizing (5) with respect to  $\mathbf{\Theta}$  for given  $\mathbf{\Lambda}$ , gives

$$\hat{\mathbf{\Theta}} = \mathbf{I} - \text{diag } \mathbf{\Lambda} \mathbf{\Lambda}'. \quad (6)$$

such that the second term in (5) vanishes. Hence, minimizing (5) is equivalent to minimizing the first term in (5) with respect to  $\mathbf{\Lambda}$ , which is the same as (1).

It is well known that if  $k > 1$  the minimizing matrix  $\mathbf{\Lambda}$  is only determined up to an orthogonal matrix. This is most easily seen from (4) for if  $\mathbf{\Lambda}$  is replaced by  $\mathbf{\Lambda}^* = \mathbf{\Lambda} \mathbf{T}$ , where  $\mathbf{T}$  is an orthogonal matrix of order  $k \times k$  satisfying  $\mathbf{T} \mathbf{T}' = \mathbf{T}' \mathbf{T}$ , then  $\mathbf{\Lambda} \mathbf{\Lambda}' = \mathbf{\Lambda}^* \mathbf{\Lambda}^{*}$ . Hence  $\mathbf{\Lambda}^*$  and  $\mathbf{\Lambda}$  minimize  $F$ . The LISREL implementation of MINRES resolves this indeterminacy by setting  $\lambda_{ij} = 0$  for  $i < k$ ,  $j > i$ , i.e., it fixes zeros in the upper right corner of  $\mathbf{\Lambda}$ . The matrix  $\mathbf{\Lambda}$  determined in this way can then be rotated to any other orthogonal or oblique solution to facilitate interpretation of the factors. Varimax and promax rotations, and a reference variables solution is included in LISREL just as in the ML case, see Jöreskog, *et al.* (2000) pp. 143–156.

Let  $\hat{\mathbf{\Lambda}}_R$  be a matrix  $\mathbf{\Lambda}$  that minimizes  $f(\mathbf{R}, \mathbf{\Lambda})$  and let  $\hat{\mathbf{\Lambda}}_S$  be a matrix  $\mathbf{\Lambda}$  that minimizes  $f(\mathbf{S}, \mathbf{\Lambda})$ . A desirable property is that

$$\hat{\mathbf{\Lambda}}_S = \mathbf{D}_s \hat{\mathbf{\Lambda}}_R, \quad (7)$$

where  $\mathbf{D}_s = (\text{diag } \mathbf{S})^{\frac{1}{2}}$  is a diagonal matrix of standard deviations. However, unlike ML, this property *does not* hold for MINRES or ULS, i.e., applying MINRES to  $\mathbf{S}$  and to  $\mathbf{R}$  does not give factor loadings that are properly related. It is therefore best to use MINRES with a correlation matrix. If it is applied to a covariance matrix, LISREL will first scale this to a correlation matrix, then estimate  $\hat{\mathbf{\Lambda}}_R$ , and then scale this by  $\mathbf{D}_s$  such that (7) will hold.

## 2 LISREL Implementation

The general LISREL syntax for exploratory factor analysis is

```
FA [NF=k] [ML] [MR]
```

where the parameters in brackets are optional. **NF=k** is used to specify the number of factors  $k$ , **ML** or **MR** are used to specify **ML** or **MINRES**, respectively. If neither **ML** nor **MR** is specified, **ML** will be used. If **NF** is not specified, the number of factors will be determined by LISREL. With **ML** this is done by a decision procedure described in Jöreskog, *et al.* (2000) pp. 153–155. With **MINRES** this is determined by “Kaiser’s little jiffy” which says that the number of factors should be equal to the number of eigenvalues of **R** which are greater than one<sup>2</sup>.

The factor analysis will be done on all variables in the data unless a subset of variables is selected by a line

```
SE varlist
```

where **varlist** is a list of variable names.

Exploratory factor analysis can also be used with **SIMPLIS** syntax. The command line is then

```
Factor Analysis [with k factors]
```

To obtain the **MINRES** solution include the line

```
Method of Estimation: MINRES
```

or

```
Method of Estimation: Unweighted Least Squares
```

Shorter versions of these are

```
Options: MR
```

or

```
Options: ULS
```

To select a subset of variables in the **SIMPLIS** command language, use<sup>3</sup>

```
Select varlist
```

If raw data is the starting point of the analysis (rather than a correlation or covariance matrix), one can use the **PRELIS** command

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<sup>2</sup>Henry Kaiser gave this rule in his dissertation 1956, see Kaiser (1970). Actually, this is the first part of the little jiffy; the second part says that the factors should be rotated by varimax.

<sup>3</sup>This is new in LISREL 8.54.

FA [NF=k] [ML] [MR] [FS]

where FS is used to specify that one wants factor scores, see Jöreskog, *et al.* (2000), pp. 155–156. These factor scores are estimated for the reference variable solution which is given at the end of the output. For details, see <http://www.ssicentral.com/lisrel/column6.htm>. The factor scores require continuous variables. However, if some of the variables are ordinal, they will be converted to normal scores (see Jöreskog, *et al.* (2000), pp. 161–168) before the factor scores are computed.

### 3 Examples

To illustrate exploratory factor analysis with MINRES I give three examples. The first example is based on the nine psychological variables used in Jöreskog, *et al.* (2000) pp. 147–156. The second example is based on the polychoric correlations of eight ordinal variables used in Jöreskog & Sörbom (2002a) pp. 111–114. The third example illustrates a Heywood case.

#### 3.1 Nine Psychological Variables

The correlation matrix is given in Table 3.4 on p. 148 in Jöreskog, *et al.* (2000). It is also given in the file **NPV.KM** in the **LIS850EX** subdirectory. Using the SIMPLIS command language (see Jöreskog & Sörbom, 2002b), one can do MINRES exploratory factor analysis of the nine psychological variables with the following input

```
Exploratory Factor Analysis of Nine Psychological Variables
Observed Variables
  'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP' WORDMEAN
  ADDITION COUNTDOT 'S-C CAPS'
Covariance Matrix from File NPV.KM
Sample Size 145
Factor Analysis
Method of Estimation: MINRES
End of Problem
```

This is the same file as **NPV2.SPL** in the **LIS850EX** subdirectory but with the line

```
Method of Estimation: MINRES
```

added.

The results are:

```
MINRES Factor Analysis for 3 Factors
```

```
Unrotated Factor Loadings
```

	Factor 1	Factor 2	Factor 3	Unique Var
	-----	-----	-----	-----
VIS PERC	0.73	0.00	0.00	0.47
CUBES	0.48	0.00	0.00	0.77
LOZENGES	0.68	0.05	-0.04	0.53
PAR COMP	0.46	0.69	0.27	0.24
SEN COMP	0.41	0.63	0.37	0.30
WORDMEAN	0.44	0.66	0.24	0.32
ADDITION	0.15	-0.11	0.77	0.37
COUNTDOT	0.40	-0.33	0.63	0.33
S-C CAPS	0.60	-0.10	0.41	0.46

Varimax-Rotated Factor Loadings

	Factor 1	Factor 2	Factor 3	Unique Var
	-----	-----	-----	-----
VIS PERC	0.69	0.19	0.13	0.47
CUBES	0.45	0.12	0.09	0.77
LOZENGES	0.65	0.21	0.07	0.53
PAR COMP	0.24	0.83	0.07	0.24
SEN COMP	0.18	0.79	0.19	0.30
WORDMEAN	0.24	0.79	0.06	0.32
ADDITION	-0.03	0.17	0.77	0.37
COUNTDOT	0.28	0.00	0.77	0.33
S-C CAPS	0.49	0.19	0.52	0.46

Promax-Rotated Factor Loadings

	Factor 1	Factor 2	Factor 3	Unique Var
	-----	-----	-----	-----
VIS PERC	0.72	0.02	-0.01	0.47
CUBES	0.48	0.01	-0.01	0.77
LOZENGES	0.68	0.06	-0.07	0.53
PAR COMP	0.07	0.85	-0.04	0.24
SEN COMP	0.00	0.81	0.09	0.30
WORDMEAN	0.08	0.80	-0.05	0.32
ADDITION	-0.19	0.13	0.81	0.37
COUNTDOT	0.20	-0.14	0.76	0.33
S-C CAPS	0.43	0.04	0.44	0.46

Factor Correlations

	Factor 1	Factor 2	Factor 3
	-----	-----	-----
Factor 1	1.00		
Factor 2	0.45	1.00	
Factor 3	0.36	0.26	1.00

Reference Variables Factor Loadings Estimated by TSLS

	Factor 1	Factor 2	Factor 3	Unique Var
	-----	-----	-----	-----
VIS PERC	0.73	0.00	0.00	0.47
CUBES	0.48 (0.21) 2.34	-0.01 (0.14) -0.04	0.00 (0.15) 0.01	0.77
LOZENGES	0.67 (0.26) 2.57	0.06 (0.14) 0.39	-0.06 (0.18) -0.32	0.53
PAR COMP	0.00	0.87	0.00	0.24
SEN COMP	-0.04 (0.15) -0.27	0.81 (0.11) 7.16	0.13 (0.11) 1.12	0.30
WORDMEAN	0.01 (0.14) 0.06	0.82 (0.11) 7.47	-0.01 (0.11) -0.10	0.32
ADDITION	0.00	0.00	0.79	0.37
COUNTDOT	0.41 (0.18) 2.26	-0.28 (0.13) -2.10	0.74 (0.21) 3.52	0.33
S-C CAPS	0.54 (0.18) 2.94	-0.05 (0.13) -0.37	0.44 (0.14) 3.12	0.46

Factor Correlations

	Factor 1	Factor 2	Factor 3
	-----	-----	-----
Factor 1	1.00		
Factor 2	0.52	1.00	
Factor 3	0.19	0.28	1.00

This may be compared with the results obtained with maximum likelihood given on pp. 149–151 in Jöreskog, *et al.* (2000), see particularly the explanations of the different results given in the bottom half of p. 151.

The first solution is the unrotated MINRES solution computed. Note that this has zeros in the upper right corner of the factor matrix. The second solution is the varimax solution

of Kaiser (1958). Both of these are orthogonal solutions, *i.e.*, the factors are uncorrelated. The third solution is the promax solution of Hendrickson & White (1964). This is an oblique solution, *i.e.*, the factors are correlated. The varimax and the promax solutions are transformations of the unrotated solution and as such they are still MINRES solutions. The fourth solution is a reference variables solution. This uses instrumental variables methods developed by Häggglund (1982) to obtain the factor loadings and the factor covariance matrix. The reference variables are chosen as those variables in the promax solution that have the largest factor loadings in each column. This gives VIS PERC, PAR COMP, and ADDITION as reference variables. The advantage of this solution is that standard error estimates can be obtained for all the variables except for the reference variables. This makes it easy to determine which loadings are statistically significant or not. The standard error estimates are given in parentheses below the loading estimate and the *t*-values are given below the standard errors. A simple rule to follow is to judge a factor loading statistically significant if its *t*-value is larger than 2 in magnitude.

The interpretation of these results is that the first factor is Visual Perception, the second is Verbal Ability, and the third is Speed. The large loadings of COUNTDOT and S-C CAPS on the first factor suggests that these variables do not measure Speed solely, like ADDITION, but also Visual Perception. This interpretation is just the same as for maximum likelihood factor analysis. For most practical purposes these two methods give the same results.

To illustrate how one can get factor scores for the factors in the reference variables solution, I use the file **NPV7C.PR2** with MR added on the FA line:

```
Computing Factor Scores
Data NI = 9
Labels
'VIS PERC' CUBES LOZENGES 'PAR COMP' 'SEN COMP'
WORDMEAN ADDITION COUNTDOT 'S-C CAPS'
Rawdata=NPV.RAW
Continuous 'VIS PERC' - 'S-C CAPS'
FA NF=3 MR FS
Output MA=KM
```

The results are the same as before. The factor scores are found in the file **NPV7C.FSC**. These factor scores may be merged with the original variables in **NPV.RAW** and may be used for any suitable purpose as if they were observed, see file **NPV7D.PR2**. Both these files are available in the **LIS850EX** subdirectory.

### 3.2 Ordinal Measures of Equality and Morality

Swedish school children in grade 9 were asked questions about their attitudes on social issues in family, school, and society. Among the questions asked were the following eight items (in free translation from Swedish).

For me, questions about ...

- HUMRGHTS human rights
- EQUALCON equal conditions for all people
- RACEPROB racial problems

EQUALVAL equal value of all people

EUTHANAS euthanasia

CRIMEPUN crime and punishment

CONSCOBJ conscientious objectors

GUILT guilt and bad conscience

are:

\_\_\_ unimportant \_\_\_ not important \_\_\_ important \_\_\_ very important

The file **DATA.EX2** in the **PR2EX** subdirectory contains data on a subsample of 200 cases. Responses to the eight questions were scored 1, 2, 3, and 4 (4 = very important). Missing values were scored 0.

The PRELIS syntax file **EX2.PR2** in the **PR2EX** subdirectory can be used to compute the matrix of polychoric correlations under pairwise deletion. With the line

FA MR

added, MINRES factor analysis can be obtained at the same time. The PRELIS syntax file then is

EXAMPLE 2: ATTITUDES OF MORALITY AND EQUALITY

DA NI=8 NO=200 MI=0 TR=PA

LA

HUMRGHTS EQUALCON RACEPROB EQUALVAL EUTHANAS CRIMEPUN CONSCOBJ GUILT

RA FI=DATA.EX2

FA MR

OU MA=PM

There is no guarantee that the matrix of polychoric correlations estimated under pairwise deletion is positive definite<sup>4</sup>. If it is not, maximum likelihood factor analysis is impossible. But MINRES still works.

PRELIS finds two factors with the following results

MINRES Factor Analysis for 2 Factors

Unrotated Factor Loadings

	Factor 1	Factor 2	Unique Var
	-----	-----	-----
HUMRGHTS	0.491	0.000	0.759
EQUALCON	0.796	-0.128	0.350
RACEPROB	0.377	0.340	0.743
EQUALVAL	0.802	0.112	0.344
EUTHANAS	0.882	-0.206	0.180
CRIMEPUN	0.390	0.451	0.645

<sup>4</sup>Positive definiteness cannot be guaranteed even if it were estimated under listwise deletion.



CONSCOBJ	0.382	0.407	0.688
GUILT	0.388	0.142	0.829

Varimax-Rotated Factor Loadings

	Factor 1	Factor 2	Unique Var
	-----	-----	-----
HUMRGHTS	0.443	0.211	0.759
EQUALCON	0.774	0.227	0.350
RACEPROB	0.194	0.469	0.743
EQUALVAL	0.676	0.446	0.344
EUTHANAS	0.885	0.193	0.180
CRIMEPUN	0.158	0.575	0.645
CONSCOBJ	0.170	0.532	0.688
GUILT	0.290	0.295	0.829

Promax-Rotated Factor Loadings

	Factor 1	Factor 2	Unique Var
	-----	-----	-----
HUMRGHTS	0.410	0.121	0.759
EQUALCON	0.782	0.040	0.350
RACEPROB	0.004	0.505	0.743
EQUALVAL	0.567	0.333	0.344
EUTHANAS	0.925	-0.034	0.180
CRIMEPUN	-0.087	0.644	0.645
CONSCOBJ	-0.054	0.589	0.688
GUILT	0.194	0.268	0.829

Factor Correlations

	Factor 1	Factor 2
	-----	-----
Factor 1	1.000	
Factor 2	0.590	1.000

Reference Variables Factor Loadings Estimated by TSLS

	Factor 1	Factor 2	Unique Var
	-----	-----	-----
HUMRGHTS	0.419	0.126	0.759
	(0.12)	(0.21)	
	3.422	0.596	
EQUALCON	0.775	0.064	0.350
	(0.12)	(0.19)	
	6.293	0.332	

RACEPROB	0.071 (0.15) 0.462	0.471 (0.28) 1.676	0.743
EQUALVAL	0.603 (0.11) 5.689	0.329 (0.19) 1.777	0.344
EUTHANAS	0.905	0.000	0.180
CRIMEPUN	0.000	0.596	0.645
CONSCOBJ	0.025 (0.17) 0.149	0.546 (0.32) 1.688	0.688
GUILT	0.227 (0.13) 1.729	0.256 (0.23) 1.111	0.829

#### Factor Correlations

	Factor 1 -----	Factor 2 -----
Factor 1	1.000	
Factor 2	0.465	1.000

I interpret this as follows. The first factor loads on HUMRGHTS, EQUALCON, EQUALVAL, and EUTHANAS. This is an Equality factor. The second factor loads on RACEPROB, CRIMEPUN, CONSCOBJ, and GUILT. This is a Morality factor. The loadings on the second factor are not statistically significant, but this maybe due to the small sample size and the ordinality of the data.

### 3.3 A Heywood Case

In a simulation study Hägglund (1982) generated many samples from a specified correlation matrix. To illustrate a Heywood case I use a covariance matrix from Hägglund's bad sample, see Jöreskog & Sörbom (1999) pp. 260–264. This covariance matrix is listed in file **EX81.COV** in the **LS8EX** subdirectory. To do a MINRES factor analysis I use file **EX81C.LS8**, from the **LS8EX** subdirectory, modified as follows:

```
Ex8.1c: Analyzing Gosta's Bad Sample
DA NI=6 NO=200 MA=KM
CM FI=EX81.COV
FA NF=2 MR
OU
```

The results show that the first variable is a Heywood case:

Unrotated Factor Loadings

	Factor 1	Factor 2	Unique Var
	-----	-----	-----
VAR 1	1.00	0.00	0.00
W_A_R_N_I_N_G: A Heywood case occurred			
VAR 2	0.45	0.48	0.56
VAR 3	0.74	0.13	0.43
VAR 4	0.43	0.41	0.65
VAR 5	0.33	0.55	0.59
VAR 6	0.14	0.20	0.94

Varimax-Rotated Factor Loadings

	Factor 1	Factor 2	Unique Var
	-----	-----	-----
VAR 1	0.96	0.28	0.00
W_A_R_N_I_N_G: A Heywood case occurred			
VAR 2	0.30	0.59	0.56
VAR 3	0.68	0.34	0.43
VAR 4	0.30	0.51	0.65
VAR 5	0.16	0.62	0.59
VAR 6	0.08	0.23	0.94

Promax-Rotated Factor Loadings

	Factor 1	Factor 2	Unique Var
	-----	-----	-----
VAR 1	0.98	0.03	0.00
W_A_R_N_I_N_G: A Heywood case occurred			
VAR 2	0.06	0.62	0.56
VAR 3	0.62	0.19	0.43
VAR 4	0.10	0.53	0.65
VAR 5	-0.11	0.71	0.59
VAR 6	-0.02	0.25	0.94

Factor Correlations

	Factor 1	Factor 2
	-----	-----
Factor 1	1.00	
Factor 2	0.62	1.00

Reference Variables Factor Loadings Estimated by TSLS

	Factor 1	Factor 2	Unique Var
VAR 1	1.00	0.00	0.00

VAR 2	0.16 (0.20) 0.81	0.57 (0.35) 1.60	0.56
VAR 3	0.66 (1.07) 0.62	0.16 (1.20) 0.13	0.43
VAR 4	0.19 (0.17) 1.10	0.47 (0.29) 1.65	0.65
VAR 5	0.00	0.64	0.59
VAR 6	0.03 (0.17) 0.15	0.23 (0.27) 0.84	0.94

#### Factor Correlations

	Factor 1 -----	Factor 2 -----
Factor 1	1.00	
Factor 2	0.52	1.00

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