

THE DIAGNOSIS AND TREATMENT OF NON-NORMALITY

1. Introduction

Several statistical methods for structural equation modeling such as the Maximum Likelihood (ML), Generalized Least Squares (GLS) and Full Information Maximum Likelihood (FIML) methods for standard and multilevel structural equation modeling assume that the data follow a multivariate Normal distribution. In practice, however, the assumption of a multivariate Normal data distribution often does not hold. Consequently, alternative methods such as Weighted Least Squares (WLS), Diagonally Weighted Least Squares (DWLS) and Robust Maximum Likelihood (RML) are recommended to fit structural equation models to these data sets. Another option would be to fit the structural equation models to the Normal Scores rather than to the original data.

Mardia (1970, 1974, 1985) and Mardia & Foster (1983) proposed numerous methods to assess the normality of the data to be analyzed. More specifically, these methods are statistical tests for marginal skewness and kurtosis as well as multivariate skewness and kurtosis. These tests as well as the Weighted Least Squares (WLS), Diagonally Weighted Least Squares (DWLS) and Robust Maximum Likelihood (RML) methods for structural equation modeling are available in LISREL 8.7 for Windows (Jöreskog & Sörbom 2004). In addition, LISREL 8.7 for Windows also includes Normal Scores. The methods of Normal Scores, WLS, DWLS and RML are described in Jöreskog et al (2001) while the normality tests are outlined in Jöreskog & Sörbom (1999).

In this note, the normality tests and the Normal Scores, WLS, and RML methods of LISREL 8.7 for Windows are illustrated by using a multivariate data set consisting of the scores of a sample of school children on nine psychological tests and a measurement model. This data set is described in the next section. The measurement model is described in section 3. Section 4 illustrates the normality tests. Thereafter, the method of Normal Scores is used to fit the measurement model to the data set. In section 6, the measurement model is fitted to the data by means of the WLS and RML methods.

2. The Data

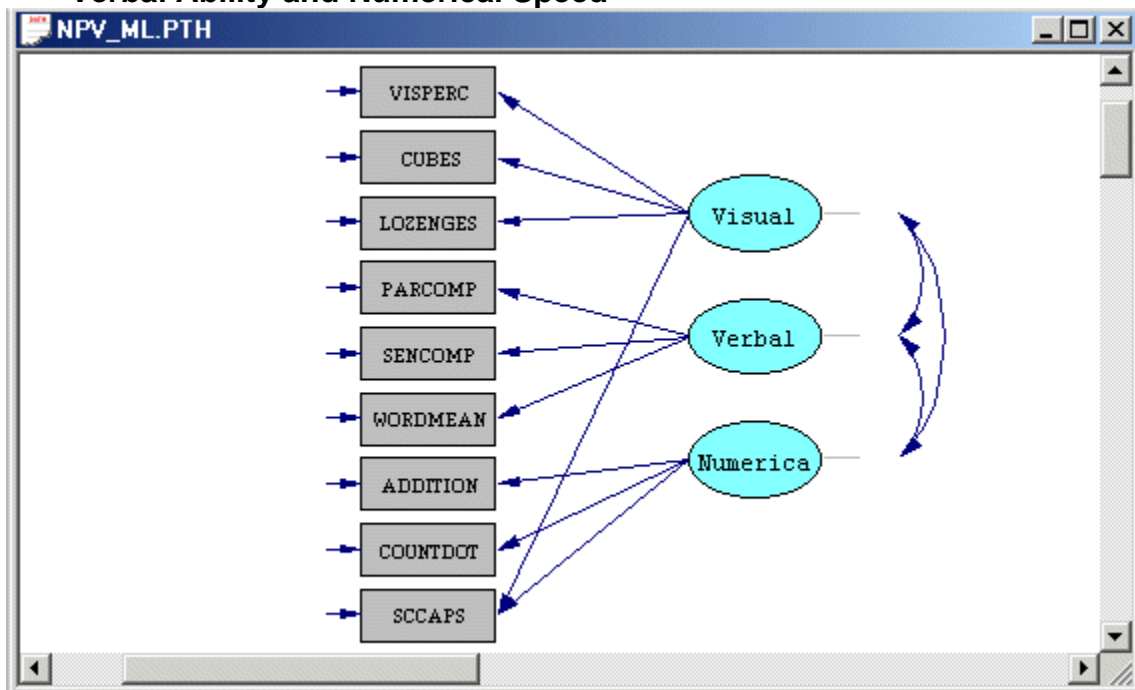
The data are the scores on nine psychological tests of 145 seventh- and eighth-grade students at the Grant-White school in Chicago. The raw data are listed in the PRELIS System File (PSF) **NPV.PSF** in the **WORKSHOP** subfolder of the LISREL 8.7 for Windows installation folder. The first couple of lines of this file are shown in the following PSF window.

	VISPERC	CUBES	LOZENGES	PARCOMP	SENCOMP	WORDMEAN	ADDITIO
1	23.000	19.000	4.000	10.000	17.000	10.000	69.0
2	33.000	22.000	17.000	8.000	17.000	10.000	65.0
3	34.000	24.000	22.000	11.000	19.000	19.000	50.0
4	29.000	23.000	9.000	9.000	19.000	11.000	114.0
5	16.000	25.000	10.000	8.000	25.000	24.000	112.0
6	30.000	25.000	20.000	10.000	23.000	18.000	94.0
7	36.000	33.000	36.000	17.000	25.000	41.000	129.0
8	28.000	25.000	9.000	10.000	18.000	11.000	96.0
9	30.000	25.000	11.000	11.000	21.000	8.000	103.0
10	20.000	25.000	6.000	9.000	21.000	16.000	89.0
11	27.000	26.000	6.000	10.000	16.000	13.000	88.0
12	32.000	21.000	8.000	1.000	7.000	11.000	103.0
13	38.000	31.000	12.000	10.000	11.000	14.000	83.0
14	17.000	21.000	6.000	5.000	10.000	10.000	99.0
15	34.000	28.000	24.000	14.000	22.000	26.000	49.0
16	25.000	31.000	18.000	7.000	12.000	11.000	65.0

3. The Measurement Model

We consider nine psychological tests that were administered to 145 seventh- and eighth-grade students. These nine psychological tests are theoretically constructed to measure the visual perception, verbal ability and numerical speed of seventh- and eighth-grade children. A path diagram for the corresponding measurement model for visual perception, verbal ability and numerical speed is shown in Figure 1.

Figure 1: A path diagram for a measurement model of Visual Perception, Verbal Ability and Numerical Speed



4. Normality Tests

- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the **WORKSHOP** folder of the LISREL 8.7 for Windows installation folder.
- Select the **PRELIS Data (*.psf)** option from the **Files of type** dropdown list box.
- Select the file **NPV.PSF**.
- Click on the **Open** push button to open the PSF window for **NPV.PSF**.
- Select the **Output Options** option on the **Statistics** menu of the PSF window to load the **Output** dialog box.
- Check the **Perform tests of multivariate normality** checkbox.
- Click on the **OK** push button of the **Output** dialog box to run PRELIS27.EXE to produce the text editor window for **NPV.OUT**.

The results for the univariate and multivariate tests of normality are shown in the following two text editor windows respectively.

Test of Univariate Normality for Continuous Variables

Variable	Skewness		Kurtosis		Skewness and Kurtosis	
	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
VISPERC	-0.604	0.546	0.045	0.964	0.367	0.833
CUBES	1.202	0.229	1.843	0.065	4.842	0.089
LOZENGES	2.958	0.003	-1.320	0.187	10.491	0.005
PARCOMP	1.995	0.046	0.761	0.447	4.559	0.102
SENCOMP	-2.646	0.008	0.693	0.489	7.483	0.024
WORDMEAN	3.385	0.001	0.720	0.472	11.977	0.003
ADDITION	0.826	0.409	-0.937	0.349	1.560	0.458
COUNTDOT	3.263	0.001	3.325	0.001	21.699	0.000
SCCAPS	1.008	0.313	1.273	0.203	2.638	0.267

Relative Multivariate Kurtosis = 1.072

Test of Multivariate Normality for Continuous Variables

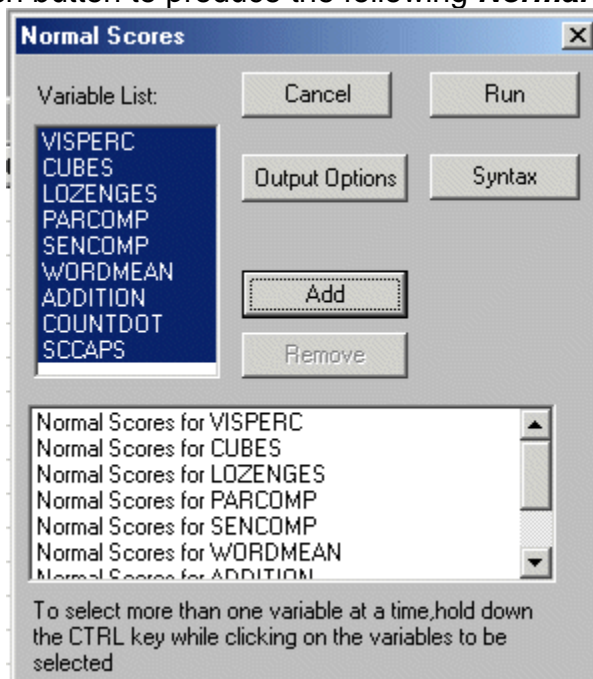
Value	Skewness		Kurtosis		Skewness and Kurtosis		
	Z-Score	P-Value	Value	Z-Score	P-Value	Chi-Square	P-Value
11.733	5.426	0.000	106.098	3.023	0.003	38.579	0.000

From the results above, it seems that there is sufficient evidence that the assumption of a multivariate Normal data distribution may be violated.

5. Using Normal Scores

Normal Scores

- Select the **Close all** option on the **Windows** menu to close all open windows.
- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the **WORKSHOP** folder of the LISREL 8.7 for Windows installation folder.
- Select the **PRELIS Data (*.psf)** option from the **Files of type** dropdown list box.
- Select the file **NPV.PSF**.
- Click on the **Open** push button to open the PSF window for **NPV.PSF**.
- Select the **Normal Scores** option on the **Statistics** menu of the PSF window to load the **Normal Scores** dialog box.
- Select all the labels in the **Variable List** box.
- Click on the **Add** push button to produce the following **Normal Scores** dialog box.



- Click on the **Output Options** push button to load the **Output** dialog box.
 - Check the **Save the transformed data to file** checkbox in the **Data** section.
 - Enter the name **NPV_NS.PSF** in the string field.
 - Click on the **OK** push button of the **Output** dialog box to return to the **Normal Scores** dialog box.
 - Click on the **Run** push button of the **Normal Scores** dialog box to run PRELIS27.EXE to produce the text editor window for **NPV.OUT**.
-
- Select the **Close all** option on the **Windows** menu to close all open windows.
 - Use the **Open** option on the **File** menu to load the **Open** dialog box.
 - Browse for the **WORKSHOP** folder of the LISREL 8.7 for Windows installation folder.
 - Select the **PRELIS Data (*.psf)** option from the **Files of type** dropdown list box.
 - Select the file **NPV_NS.PSF**.
 - Click on the **Open** push button to open the PSF window for **NPV_NS.PSF**.
 - Select the **Define Variables** option on the **Data** menu to load the **Define Variables** dialog box.
 - Select all the labels in the list box.
 - Click on the **Missing Values** push button to load the **Missing Values for ...** dialog box.
 - Activate the **Listwise Deletion** radio button.
 - Select the **Output Options** option on the **Statistics** menu of the PSF window to load the **Output** dialog box.

- Check the **Perform test of multivariate normality** checkbox.
- Click on the **OK** push button of the **Output** dialog box to run PRELIS27.EXE to produce the text editor window for **NPV_NS.OUT**.

The results for the univariate and multivariate tests of normality are shown in the following two text editor windows respectively.

Test of Univariate Normality for Continuous Variables

Variable	Skewness		Kurtosis		Skewness and Kurtosis	
	Z-Score	P-Value	Z-Score	P-Value	Chi-Square	P-Value
VISPERC	0.019	0.985	0.067	0.946	0.005	0.998
CUBES	-0.011	0.991	-0.005	0.996	0.000	1.000
LOZENGES	0.055	0.956	0.000	1.000	0.003	0.998
PARCOMP	0.016	0.988	0.119	0.905	0.014	0.993
SENCOMP	-0.038	0.969	0.096	0.924	0.011	0.995
WORDMEAN	0.025	0.980	0.092	0.927	0.009	0.995
ADDITION	0.000	1.000	0.108	0.914	0.012	0.994
COUNTDOT	0.003	0.998	0.106	0.916	0.011	0.994
SCCAPS	0.000	1.000	0.108	0.914	0.012	0.994

Relative Multivariate Kurtosis = 1.018

Test of Multivariate Normality for Continuous Variables

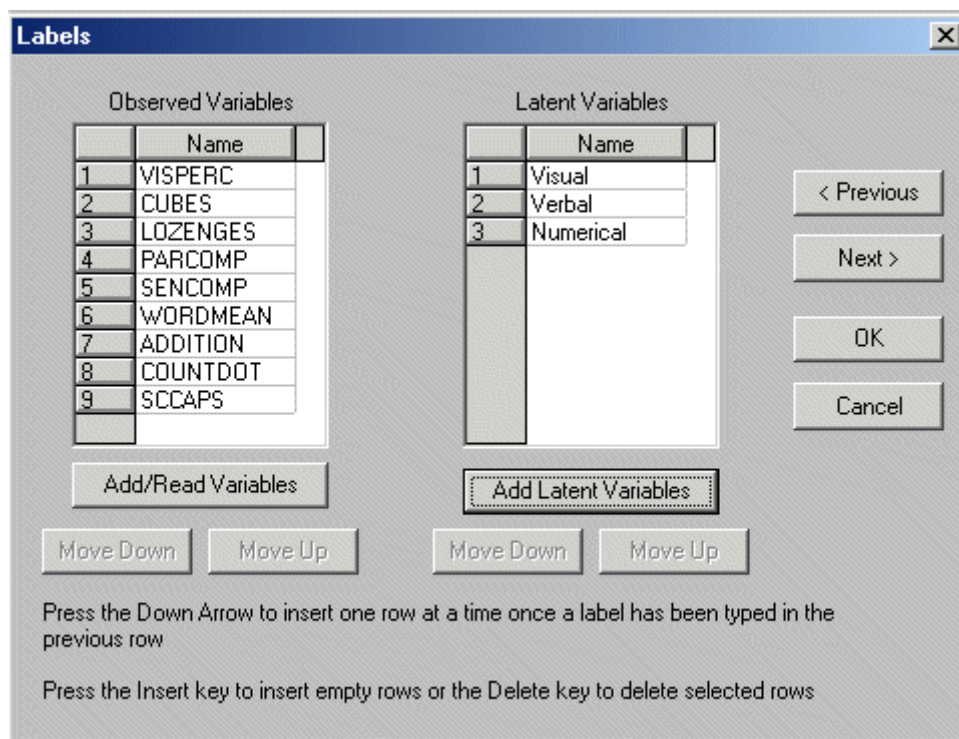
Value	Skewness		Kurtosis		Skewness and Kurtosis		
	Z-Score	P-Value	Value	Z-Score	P-Value	Chi-Square	P-Value
8.187	1.737	0.082	100.807	1.318	0.188	4.754	0.093

From the results above, it seems that there is insufficient evidence that the assumption of a multivariate Normal data distribution may be violated.

Fitting the Measurement Model to the Normal Scores

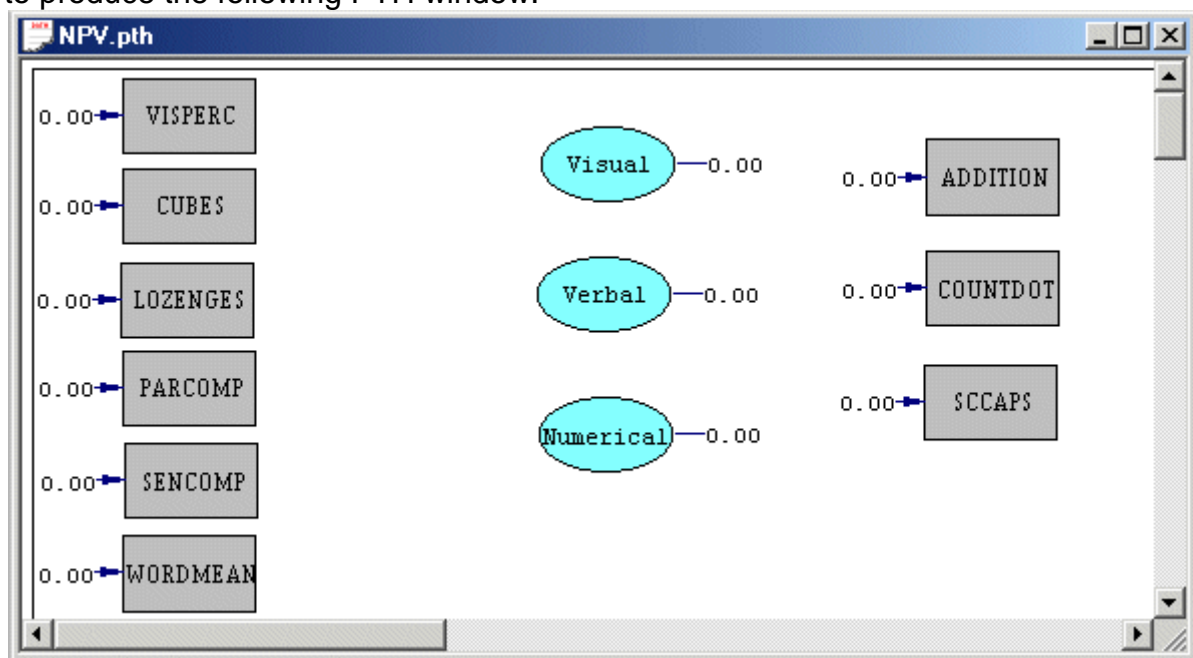
- Select the **Close all** option on the **Windows** menu to close all open windows.

- Use the **New** option on the **File** menu of the root window to load the **New** dialog box.
- Select the **Path Diagram** option from the list box on the **New** dialog box to load the **Save As** dialog box.
- Enter **NPV.PTH** in the **File name** string field.
- Click on the **Save** push button to open an empty path diagram window.
- Select the **Title and Comments** option on the **Setup** menu to load the **Title and Comments** dialog box.
- Enter **A measurement model for nine psychological tests** in the **Title** string field.
- Click on the **Next** push button to load the **Group Names** dialog box.
- Click on the **Next** push button to load the **Labels** dialog box.
- Click on the **Add/Read Variables** push button to load the **Add/Read Variables** dialog box.
- Select the **PRELIS System File** option in the **Read from file:** dropdown list box.
- Click on the **Browse** push button to load the **Browse** dialog box.
- Select the file **NPV_NS.PSF**.
- Click on the **Open** push button to return to the **Add/Read Variables** dialog box.
- Click on the **OK** push button to return to the **Labels** dialog box.
- Click on the **Add Latent Variables** push button to load the **Add Variables** dialog box.
- Enter the label **Visual** for visual perception in the string field.
- Click on the **OK** push button to return to the **Labels** dialog box.
- Click on the **Add Latent Variables** push button to load the **Add Variables** dialog box.
- Enter the label **Verbal** for verbal ability in the string field.
- Click on the **Add Latent Variables** push button to load the **Add Variables** dialog box.
- Enter the label **Numerical** for numerical speed in the string field.
- Click on the **OK** push button to produce the following **Labels** dialog box.

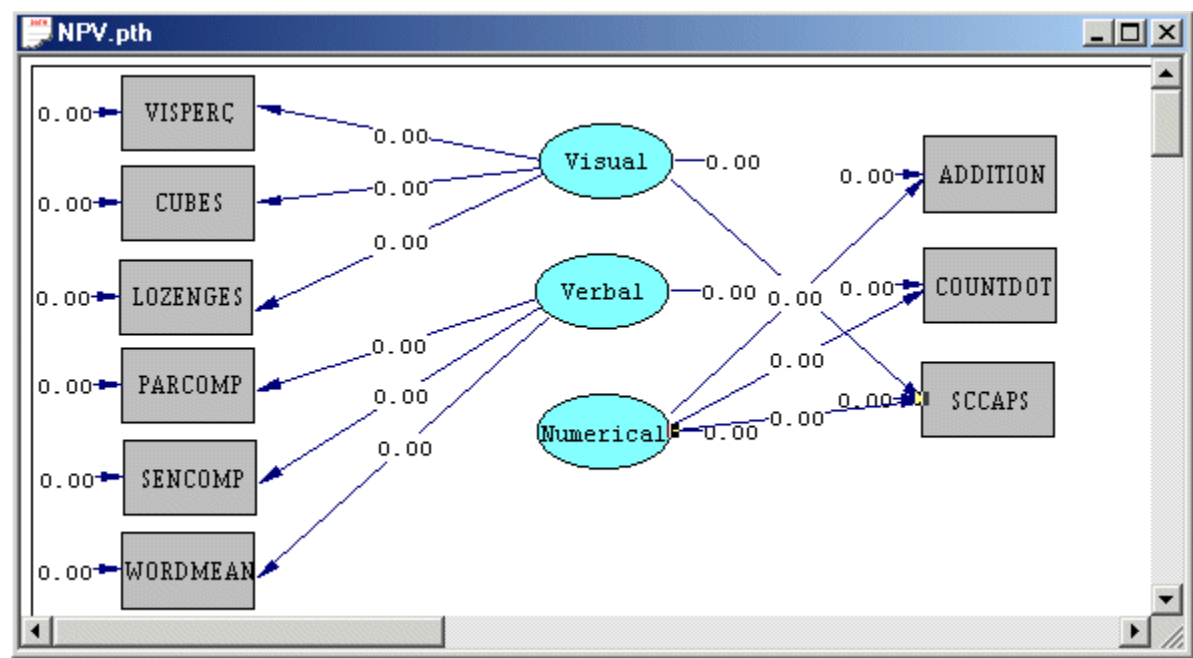


- Click on the **OK** push button to return to the PTH window for **NPV.PTH**.
- Click, drag and drop the labels of the observed variables one at a time into the PTH window.

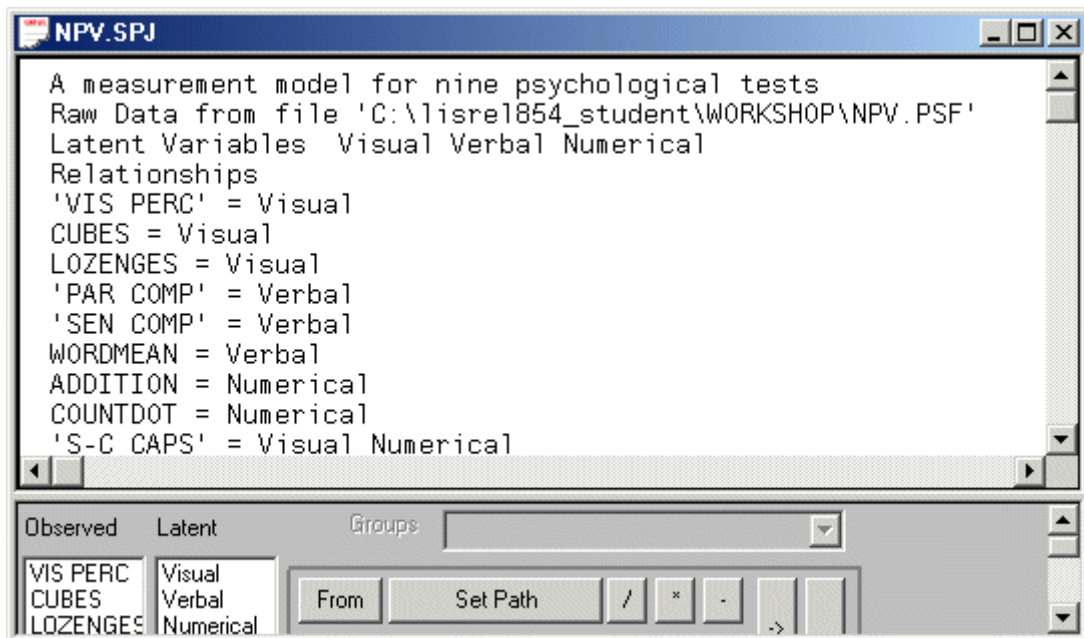
- Click, drag and drop the labels of the latent variables one at a time into the PTH window to produce the following PTH window.



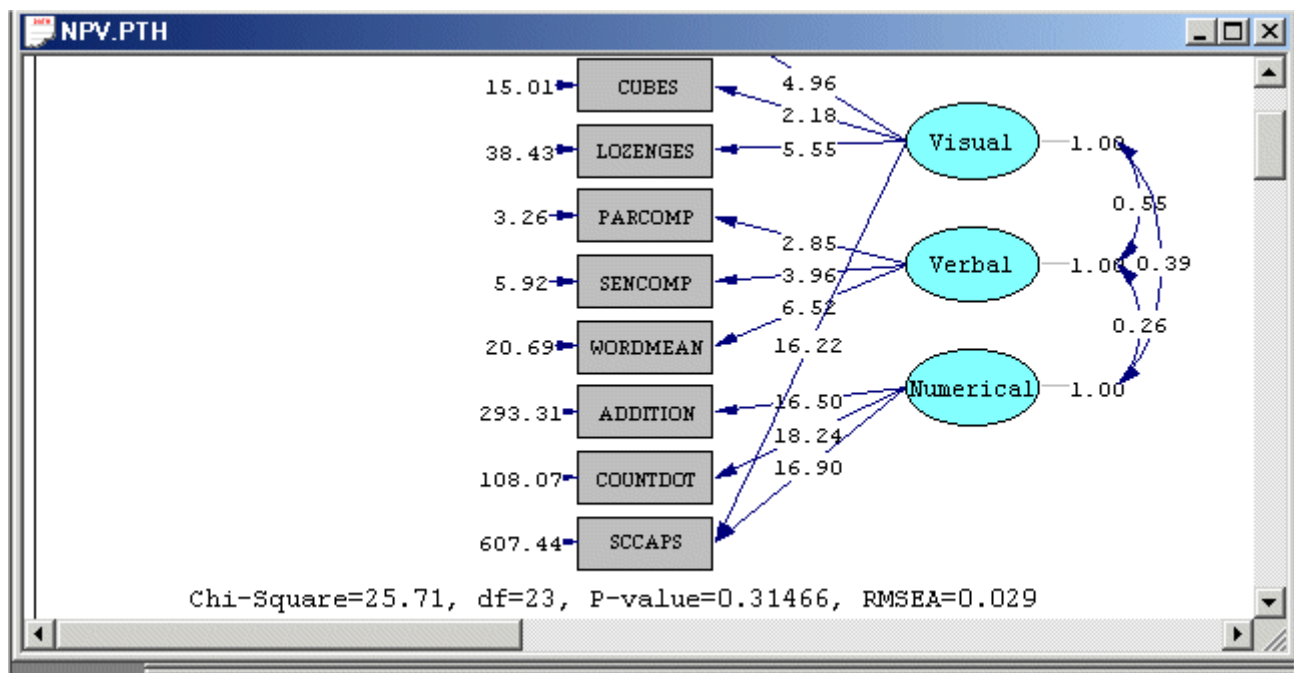
- Select the arrow icon on the drawing toolbar.
- Click and drag paths from **Visual** to **VISPERC**, **CUBES**, **LOZENGES** and **SCCAPS** respectively.
- Click and drag paths from **Verbal** to **PARCOMP**, **SENCOMP** and **WORDMEAN** respectively.
- Click and drag paths from **Numerical** to **ADDITION**, **COUNTDOT** and **SCCAPS** respectively to produce the following PTH window.



- Select the **Build SIMPLIS Syntax** option on the **Setup** menu to open the following SIMPLIS project window.



- Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.

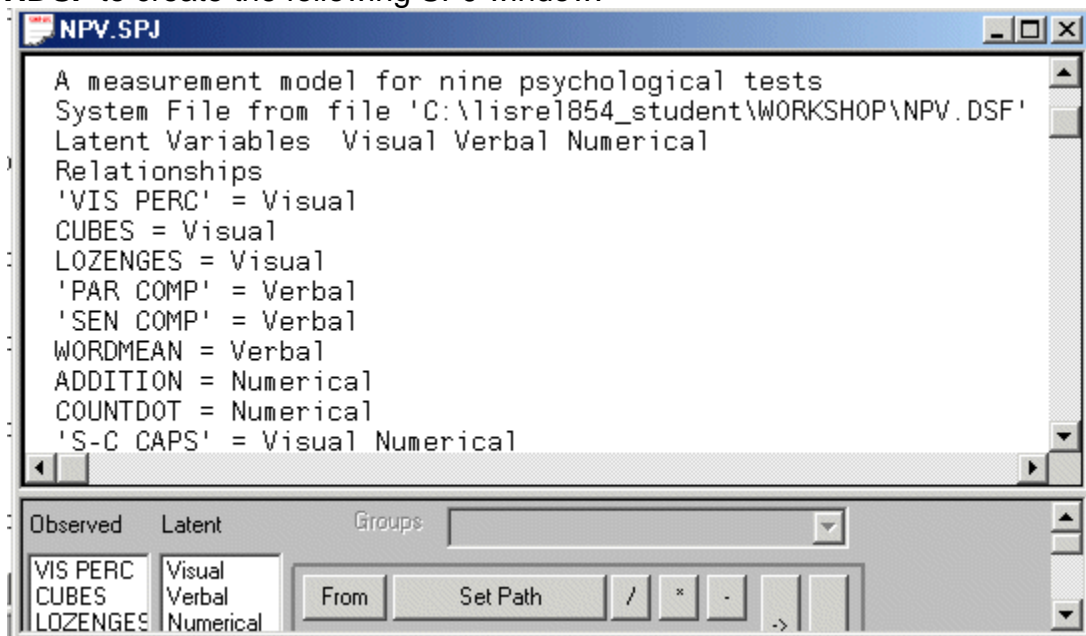


6. The RML and WLS Methods Generating the Data System File (DSF)

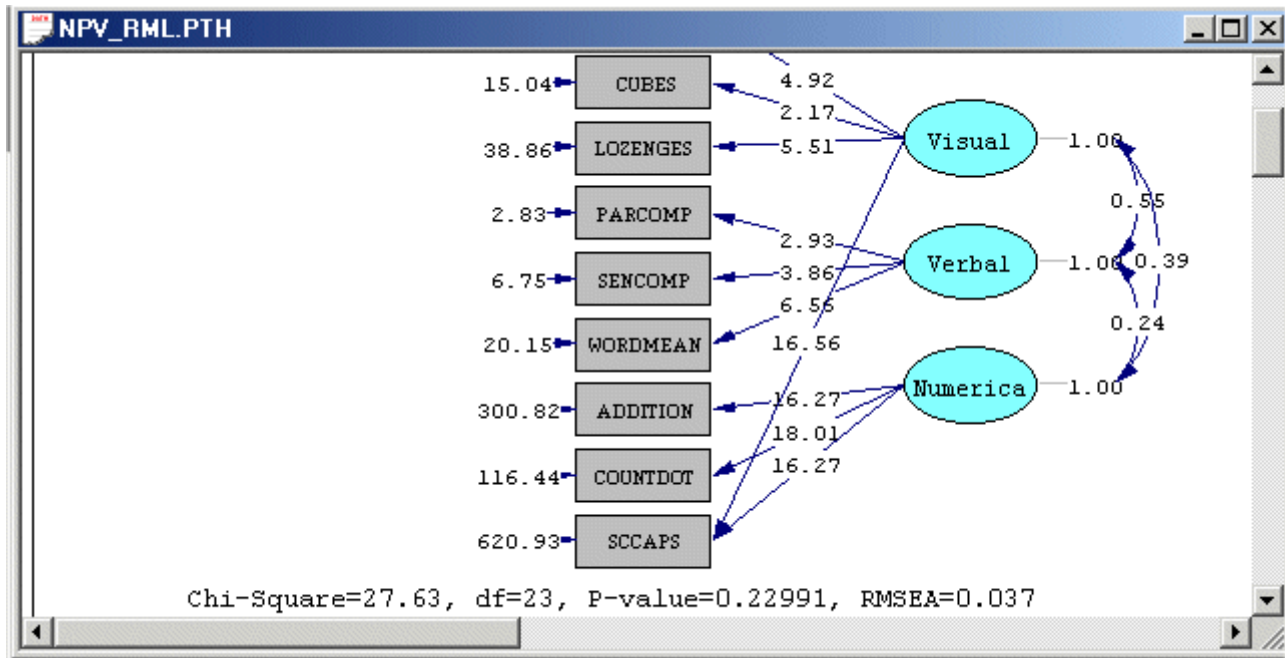
- Select the **Close all** option on the **Windows** menu to close all open windows.
- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the **WORKSHOP** folder of the LISREL 8.7 for Windows installation folder.
- Select the **PRELIS Data (*.psf)** option from the **Files of type** dropdown list box.
- Select the file **NPV.PSF**.
- Click on the **Open** push button to open the PSF window for **NPV.PSF**.
- Select the **Output Options** option on the **Statistics** menu to load the **Output** dialog box.
- Check the **Save to File** checkbox in the **Asymptotic Covariance Matrix** section.
- Enter the name **NPV.ACM** in the string field in the **Asymptotic Covariance Matrix** section.
- Click on the **OK** push button to run PRELIS27.EXE to generate the text editor window containing the output file **NPV.OUT**.

The RML Method

- Select the **Close all** option on the **Windows** menu to close all open windows.
- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the **WORKSHOP** folder of the LISREL 8.7 for Windows installation folder.
- Select the **SIMPLIS Project (*.spj)** option from the **Files of type** dropdown list box.
- Select the file **NPV.SPJ**.
- Click on the **Open** push button to open the SPJ window for **NPV.SPJ**.
- Use the **Save As** option on the **File** menu of the root window to load the **Save As** dialog box.
- Enter the name **NPV_RML.SPJ** in the **File name** string field.
- Click on the **Save** push button to open the SPJ window for **NPV_RML.SPJ**.
- Change the second line **Raw Data from File NPV_NS.PSF** to **System File from File NPV.DSF** to create the following SPJ window.



- Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



The WLS Method

- Select the **Close all** option on the **Windows** menu to close all open windows.
- Use the **Open** option on the **File** menu to load the **Open** dialog box.
- Browse for the **WORKSHOP** folder of the LISREL 8.7 for Windows installation folder.
- Select the **SIMPLIS Project (*.spj)** option from the **Files of type** dropdown list box.
- Select the file **NPV_RML.SPJ**.
- Click on the **Open** push button to open the SPJ window for **NPV_RML.SPJ**.
- Use the **Save As** option on the **File** menu of the root window to load the **Save As** dialog box.
- Enter the name **NPV_WLS.SPJ** in the **File name** string field.
- Click on the **Save** push button to open the SPJ window for **NPV_WLS.SPJ**.
- Insert the line **Method of Estimation: Weighted Least Squares** just before the **Path Diagram** command to create the following SPJ window.

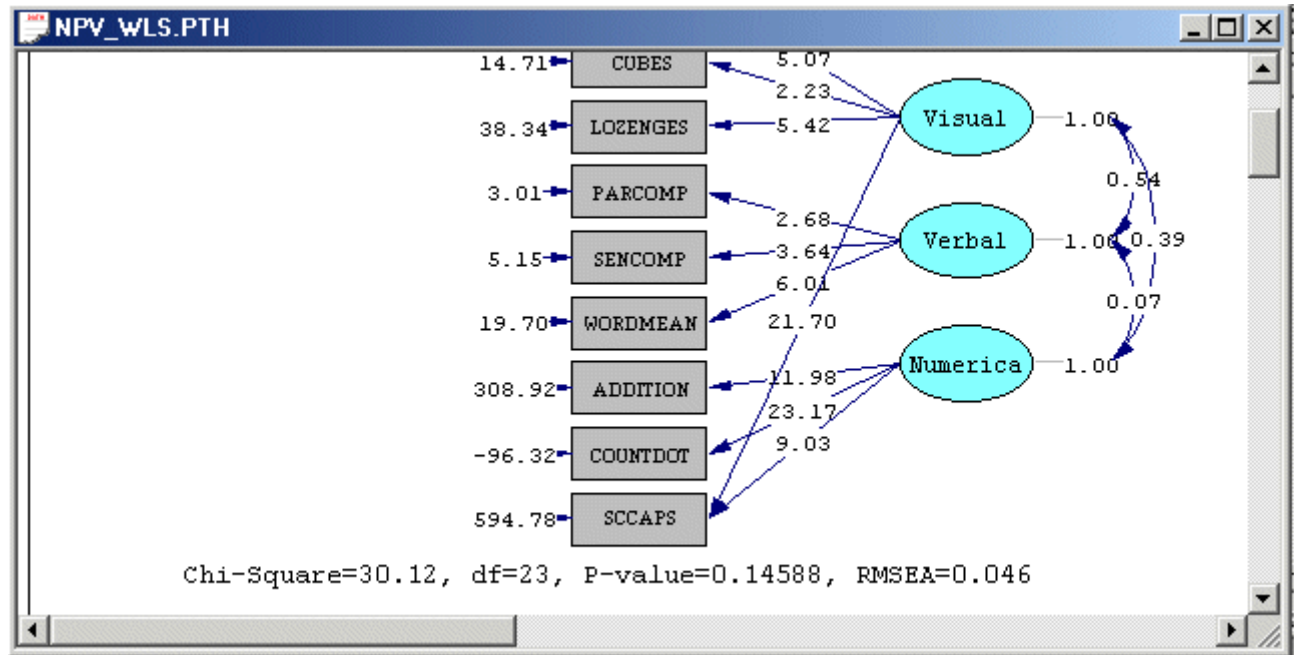
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Relationships
VISPERC = Visual
CUBES = Visual
LOZENGES = Visual
PARCOMP = Verbal
SENCOMP = Verbal
WORDMEAN = Verbal
ADDITION = Numerical
COUNTDOT = Numerical
SCCAPS = Visual Numerical
Method of Estimation: Weighted Least Squares
Path Diagram
End of Problem

```

Observed	Latent	Groups
VISPERC	Visual	
CUBES	Verbal	
LOZENGES	Numerica	

- Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.



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