Multiple group models for ordinal variables

1. Introduction

In practice, many multivariate data sets consist of observations of ordinal variables rather than continuous variables. Most statistical methods for structural equation modeling assume that the data are observations of continuous variables. In practice, this problem is overcome by treating the ordinal variables as if they were continuous variables. Unfortunately, this approach may yield misleading results (see e.g. Jöreskog 1994 and Jöreskog & Sörbom 1999). In addition, many of the multivariate data sets of ordinal variables are observations from several groups. Examples of these groups are genders, languages, political parties, countries, faculties, colleges, schools, etc. For these data sets, it is often of interest to determine whether or not the grouping variable has any influence on the structural equation model for the observed variables. The statistical methods for multiple group structural equation modeling with ordinal variables may be used to determine whether or not the grouping variable has an influence on the model.

LISREL 8.80 for Windows (Jöreskog & Sörbom 2006) includes several statistical methods for the analysis of ordinal variables. These methods are logistic regression, PROBIT regression as well as Weighted Least Squares (WLS) and Diagonally Weighted Least Squares (DWLS) for polychoric correlation and covariance matrices. In addition, it includes the DWLS and WLS methods for fitting multiple group models to multiple group data of ordinal variables. More details about these methods are provided in Jöreskog (2002).

In this note, LISREL 8.80 for Windows is used to fit a measurement model to observations on ordinal variables obtained from two countries. These two data sets are described in the next section. The measurement model is described in Section 3. The measurement model is fitted to the two data sets of ordinal variables in Section 4. In Section 5, the equality of the factor loadings across the two countries is assessed.

2. The data

Barnes & Kaase (1979) conducted a cross-national survey to obtain information on conventional and unconventional forms of political participation in industrial societies. This survey is known as the Political Action Survey. This survey included several attitude and perception statements to which the respondents had to respond to as one of agree strongly, agree, disagree, disagree strongly, don't know or no answer. Amongst others, the following statements were administered to each respondent.

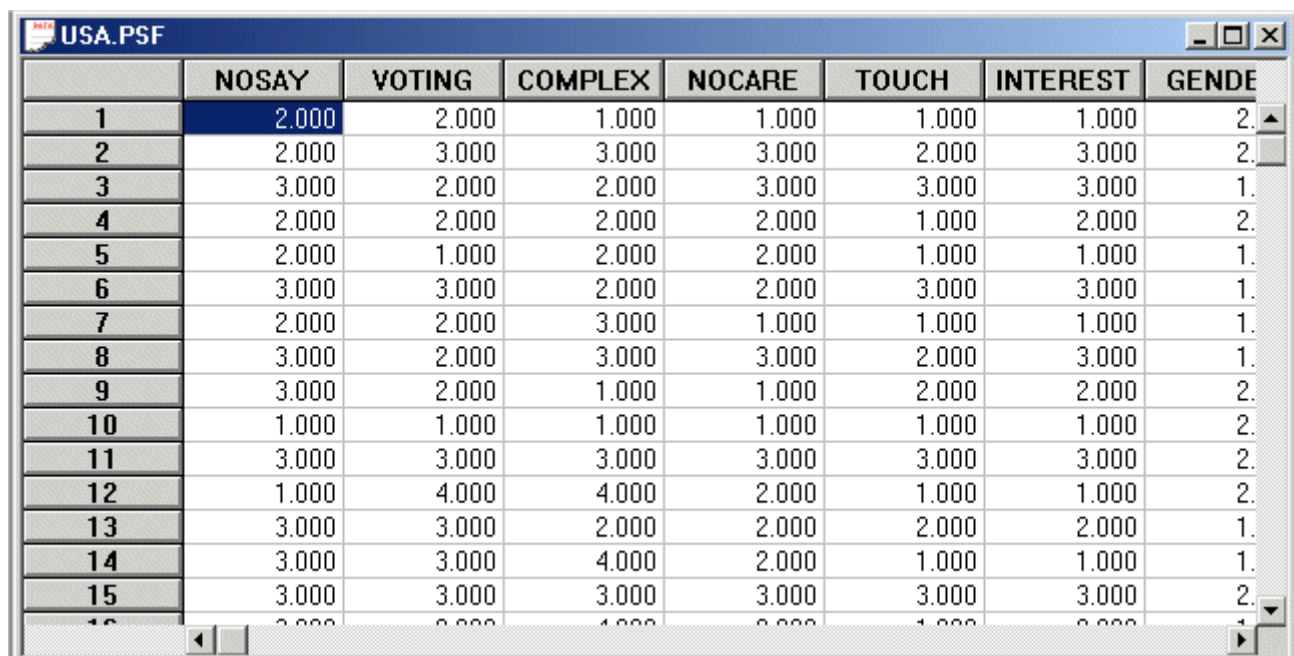*"People like me have no say in what the government does."* (**NOSAY**)

"Sometimes politics and government seem so complicated that a person like me cannot really understand what is going on." (**COMPLEX**)

"I don't think that public officials care much about what people like me think." (**NOCARE**)

"Generally speaking, those we elect to Congress in Washington lose touch with people pretty quickly." (**TOUCH**)

"Parties are only interested in people's votes but not in their opinions. (**INTEREST**)

The responses to, amongst others, these statements for a sample of 1076 United States respondents and 1266 British respondents are listed in the PRELIS System Files (PSF) **USA.PSF** and **BRITIAN.PSF** respectively. These files are located in the location **C:\LISREL 8.8 Student Examples\WORKSHOP**. Small portions of these files are shown in the following two PSF windows.

USA.PSF	NOSAY	VOTING	COMPLEX	NOCARE	TOUCH	INTEREST	GENDE
1	2.000	2.000	1.000	1.000	1.000	1.000	2.
2	2.000	3.000	3.000	3.000	2.000	3.000	2.
3	3.000	2.000	2.000	3.000	3.000	3.000	1.
4	2.000	2.000	2.000	2.000	1.000	2.000	2.
5	2.000	1.000	2.000	2.000	1.000	1.000	1.
6	3.000	3.000	2.000	2.000	3.000	3.000	1.
7	2.000	2.000	3.000	1.000	1.000	1.000	1.
8	3.000	2.000	3.000	3.000	2.000	3.000	1.
9	3.000	2.000	1.000	1.000	2.000	2.000	2.
10	1.000	1.000	1.000	1.000	1.000	1.000	2.
11	3.000	3.000	3.000	3.000	3.000	3.000	2.
12	1.000	4.000	4.000	2.000	1.000	1.000	2.
13	3.000	3.000	2.000	2.000	2.000	2.000	1.
14	3.000	3.000	4.000	2.000	1.000	1.000	1.
15	3.000	3.000	3.000	3.000	3.000	3.000	2.

	NOSAY	VOTING	COMPLEX	NOCARE	TOUCH	INTEREST	
1	2.000	2.000	2.000	2.000	2.000	2.000	
2	2.000	2.000	2.000	2.000	2.000	2.000	
3	2.000	2.000	2.000	2.000	2.000	2.000	
4	2.000	1.000	2.000	2.000	2.000	2.000	
5	3.000	3.000	2.000	3.000	3.000	3.000	
6	2.000	2.000	2.000	3.000	3.000	3.000	
7	2.000	2.000	1.000	2.000	2.000	2.000	
8	2.000	2.000	2.000	2.000	2.000	2.000	
9	3.000	3.000	3.000	2.000	3.000	3.000	
10	1.000	2.000	2.000	2.000	3.000	3.000	
11	2.000	2.000	1.000	2.000	2.000	2.000	
12	3.000	1.000	3.000	3.000	2.000	3.000	
13	2.000	2.000	2.000	3.000	2.000	2.000	
14	2.000	3.000	2.000	3.000	3.000	3.000	
15	2.000	2.000	2.000	2.000	2.000	2.000	

BRITAIN.PSF

3. The measurement model

The five political statements, which are listed in Section 2, are regarded as indicators of political efficacy and political responsiveness. More specifically, *NOSAY*, *COMPLEX*, and *NOCARE* are indicators of political efficacy and *TOUCH* and *INTEREST* are indicators of political responsiveness. A path diagram of the corresponding measurement model is shown in Figure 1.

Figure 1: A path diagram for a measurement model for political efficacy and responsiveness
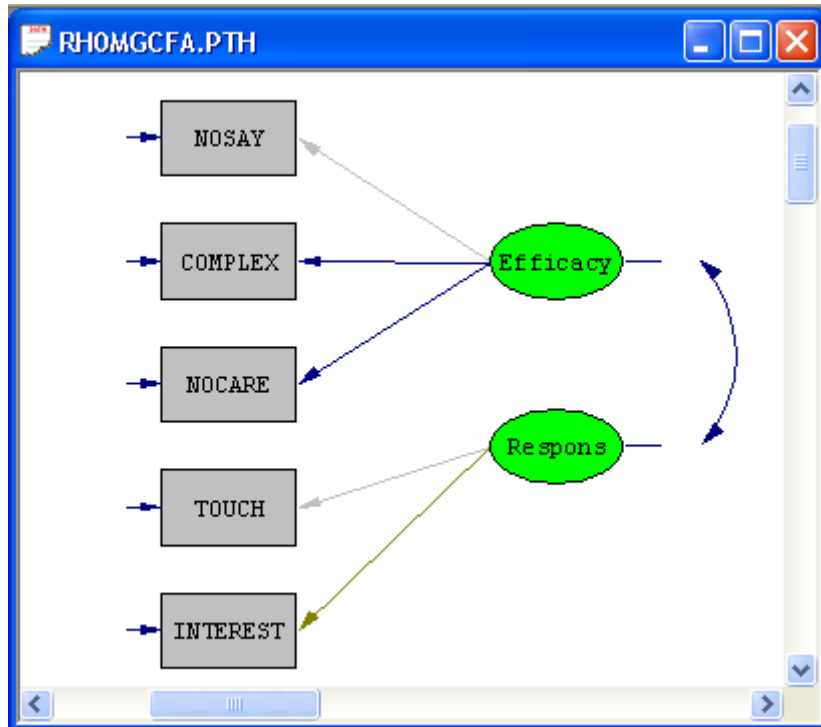
4. Fitting the measurement model to the USA and Britain data

Generating the matrix files for the USA data

➢ Use the *Open* option on the *File* menu to load the *Open* dialog box.
➢ Browse for the location *C:\LISREL 8.8 Student Examples\WORKSHOP*.
➢ Select the file *USA1.PR2* by clicking on it.
➢ Click on the *Open* button to open the following text editor window.


```
Computing matrices for country USA
SY=USA.PSF
FT=EFFITOT.THR NOSAY
FT VOTING
FT COMPLEX
FT NOCARE
FT TOUCH
FT INTEREST
OU MA=CM CM=USA.COV AC=USA.ACM
```

Line 1 provides a title for the computation.
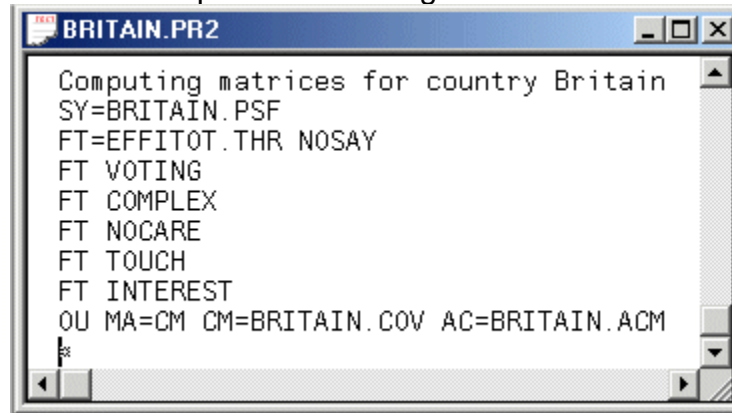Line 2 specifies the raw data source.
Lines 3 to 8 specify that the matrices should be computed under the assumption of fixed thresholds for **NOSAY**, **VOTING**, **COMPLEX**, **NOCARE**, **TOUCH**, and **INTEREST**.

Line 9 specifies that the polychoric covariance matrix should be computed and written to the text file **USA.COV** and that the estimated asymptotic covariance matrix of the polychoric variances and covariances should be computed and written to the binary file **USA.ACM**.

➢ Click on the **OK** button to generate the text editor window for **USA1.OUT**.

Generating the matrix files for the Britain data
➢ Use the **Open** option on the **File** menu to load the **Open** dialog box.
➢ Browse for the location **C:\LISREL 8.8 Student Examples\WORKSHOP.**
➢ Select the file **BRITAIN.PR2** by clicking on it.
➢ Click on the **Open** button to open the following text editor window.


```
BRITAIN.PR2                          _|□|×|
Computing matrices for country Britain
SY=BRITAIN.PSF
FT=EFFITOT.THR NOSAY
FT VOTING
FT COMPLEX
FT NOCARE
FT TOUCH
FT INTEREST
OU MA=CM CM=BRITAIN.COV AC=BRITAIN.ACM
*
```

Line 1 provides a title for the computation.
Line 2 specifies the raw data source.
Lines 3 to 8 specify that the matrices should be computed under the assumption of fixed thresholds for **NOSAY**, **VOTING**, **COMPLEX**, **NOCARE**, **TOUCH**, and **INTEREST**.
Line 9 specifies that the polychoric covariance matrix should be computed and written to the text file **BRITAIN.COV** and that the estimated asymptotic covariance matrix of the polychoric variances and covariances should be computed and written to the binary file **BRITAIN.ACM**.

➢ Click on the **OK** button to generate the text editor window for **BRITAIN.OUT**.

Fitting the measurement model to the two data sets of ordinal variables
➢ Use the **Open** option on the **File** menu to load the **Open** dialog box.
➢ Browse for the location **C:\LISREL 8.8 Student Examples\WORKSHOP.**
➢ Select the file **H1MGCFA.SPL** by clicking on it.
➢ Click on the **Open** button to open the following text editor window.

```
H1MGCFA.SPL

Group 1: USA
Observed Variables: NOSAY VOTING COMPLEX NOCARE TOUCH INTEREST
Covariance Matrix from File USA.COV
Asymptotic Covariance Matrix from File USA.ACM
Sample Size: 1076
Latent Variables: Efficacy Respons
NOSAY =   1*Efficacy
COMPLEX NOCARE = Efficacy
TOUCH = 1*Respons
INTEREST = Respons
Group 2: BRITAIN
Observed Variables: NOSAY VOTING COMPLEX NOCARE  TOUCH INTEREST
Covariance Matrix from File BRITAIN.COV
Asymptotic Covariance Matrix from File BRITAIN.ACM
Sample Size: 1266
Latent Variables: Efficacy Respons
NOSAY =   1*Efficacy
COMPLEX NOCARE = Efficacy
TOUCH = 1*Respons
INTEREST = Respons
Set the Variance of Efficacy - Respons Free
Set the Covariance between Efficacy and Respons
Set the Error Variance of NOSAY Free
Set the Error Variance of COMPLEX - INTEREST Free
LISREL Output: ND=3 SC ME=WLS AD=OFF
Path Diagram
End of Problem
```

Line 1 provides a title for the USA group.
Line 2 specifies labels for the observed variables.
Lines 3 and 4 specify the matrix files for the polychoric covariance matrix and the estimated asymptotic covariance matrix of the polychoric variances and covariances respectively.
Line 5 specifies the number of observations in USA.PSF.
Line 6 specifies labels for the latent variables.
Lines 7 to 10 specify the measurement model for the USA group.
Line 11 provides a title for the Britain group.
Line 12 specifies labels for the observed variables.
Lines 13 and 14 specify the matrix files for the polychoric covariance matrix and the estimated asymptotic covariance matrix of the polychoric variances and covariances respectively.
Line 15 specifies the number of observations in BRITAIN.PSF.
Line 16 specifies labels for the latent variables.
Lines 17 to 24 specify the measurement model for the Britain group.
Line 25 specifies the multiple group Weighted Least Squares (WLS) method for ordinal variables and that the results should be printed in terms of the LISREL model for the measurement model in Figure 1.
Line 26 requests a path diagram.
Line 27 indicates no more SIMPLIS commands are to be processed.

- ➢ Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.
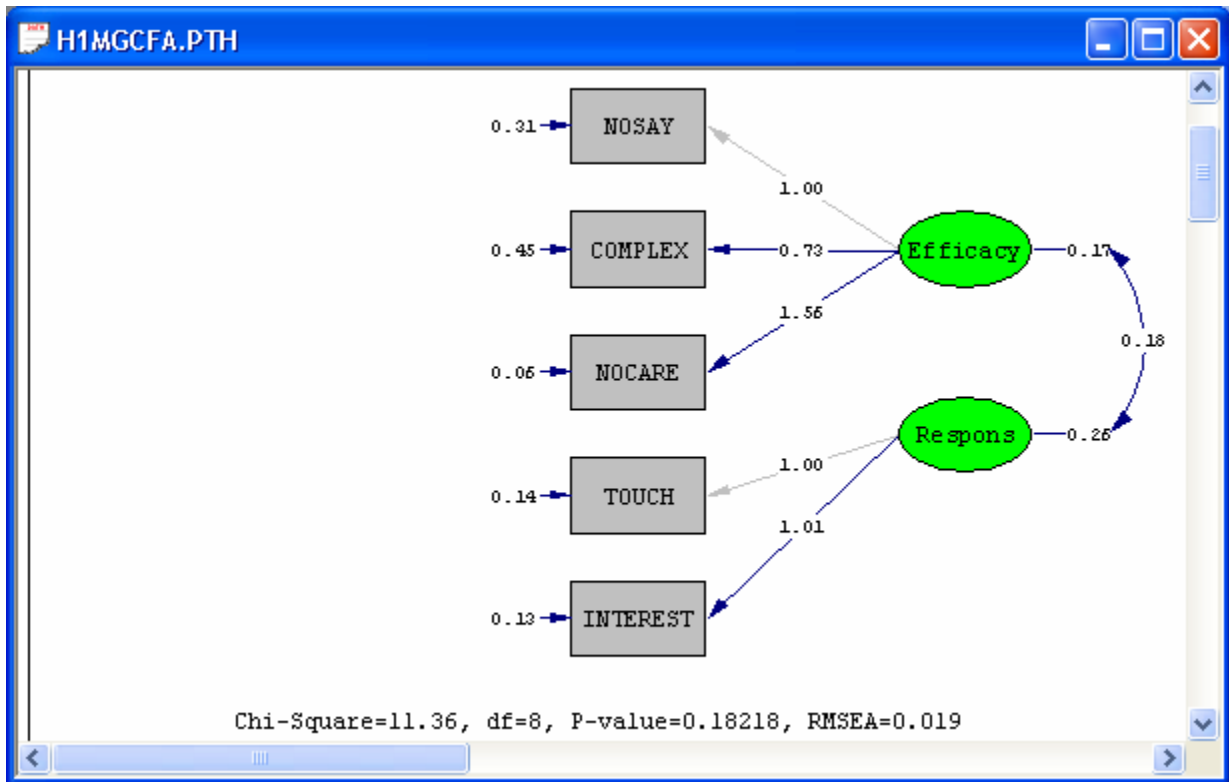
5. Invariance of factor loadings

Invariance of all factor loadings

The measurement model in Figure 1 will now be used to illustrate how the multiple-group feature for ordinal variables of LISREL 8.80 for Windows may be used to assess whether or not the factor loadings of the measurement model are invariant across the two countries. This assessment consists of testing the null hypothesis (H0) that the factor loadings are identical across the two countries against the alternative hypothesis (H1) that the factor loadings are not identical across the two countries. A Chi-square difference test is used to test H0 and H1.

The multiple-group model under H1 was already fitted to the data from the two countries in section 4. The multiple-group model under H0 may be fitted as follows.

Fitting the multiple group model under H0
- ➢ Use the **Open** option on the **File** menu to load the **Open** dialog box.
- ➢ Browse for the location **C:\LISREL 8.8 Student Examples\WORKSHOP.**
- ➢ Select the file **H0MGCFA.SPL** by clicking on it.
- ➢ Click on the **Open** button to open the following text editor window.

```
HOMGCFA.SPL

    Group 1: USA
    Observed Variables: NOSAY VOTING COMPLEX NOCARE TOUCH INTEREST
    Covariance Matrix from File USA.COV
    Asymptotic Covariance Matrix from File USA.ACM
    Sample Size: 1076
    Latent Variables: Efficacy Respons
    NOSAY =  1*Efficacy
    COMPLEX NOCARE = Efficacy
    TOUCH = 1*Respons
    INTEREST = Respons
    Group 2: BRITAIN
    Observed Variables: NOSAY VOTING COMPLEX NOCARE  TOUCH INTEREST
    Covariance Matrix from File BRITAIN.COV
    Asymptotic Covariance Matrix from File BRITAIN.ACM
    Sample Size: 1266
    Latent Variables: Efficacy Respons
    Set the Variance of Efficacy - Respons Free
    Set the Covariance between Efficacy and Respons
    Set the Error Variance of NOSAY Free
    Set the Error Variance of COMPLEX - INTEREST Free
    LISREL Output: ND=3 SC ME=WLS AD=OFF
    Path Diagram
    End of Problem
    L
```

Line 1 provides a title for the USA group.
Line 2 specifies labels for the observed variables.
Lines 3 and 4 specify the matrix files for the polychoric covariance matrix and the estimated asymptotic covariance matrix of the polychoric variances and covariances respectively.
Line 5 specifies the number of observations in USA.PSF.
Line 6 specifies labels for the latent variables.
Lines 7 to 10 specify the measurement model for the USA group.
Line 11 provides a title for the Britain group.
Line 12 specifies labels for the observed variables.
Lines 13 and 14 specify the matrix files for the polychoric covariance matrix and the estimated asymptotic covariance matrix of the polychoric variances and covariances respectively.
Line 15 specifies the number of observations in BRITAIN.PSF.
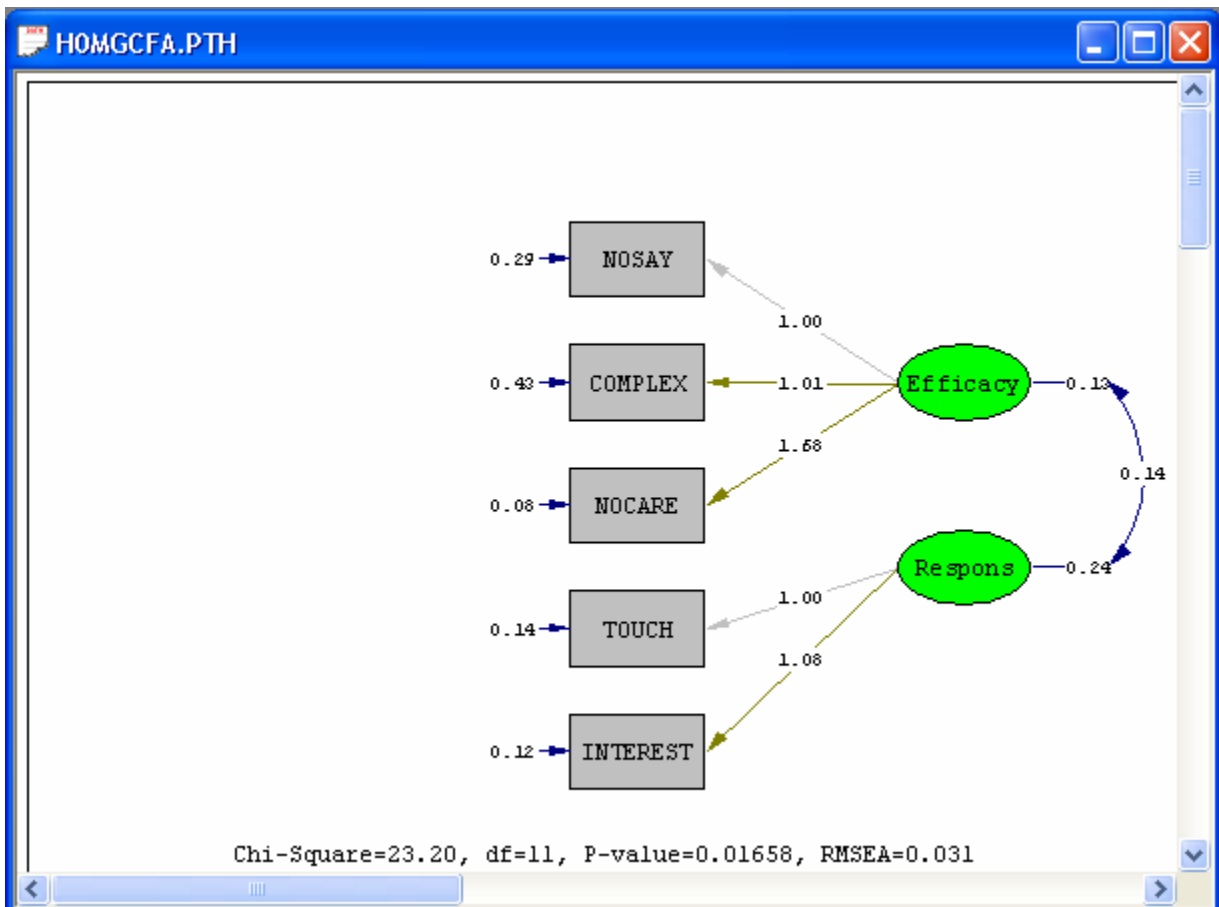Line 16 specifies labels for the latent variables.
Lines 17 to 20 specify the measurement model for the Britain group.
Line 21 specifies the Weighted Least Squares (WLS) method for ordinal variables and that the results should be printed in terms of the LISREL model for the measurement model in Figure 1.
Line 22 requests a path diagram.
Line 23 indicates no more SIMPLIS commands are to be processed.

➢ Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.

Multiple group variables for ordinal variables   8

Performing the Chi-square difference test

A Chi-square difference test is used to assess whether or not the factor loadings of the measurement model are invariant across the two countries. In other words, a Chi-square difference test is used to test the null and alternative hypotheses. The test statistic value for the Chi-square difference test is merely the difference between the goodness-of-fit Chi-square test statistic values of the multiple group measurement models under the null and the alternative hypotheses. The associated degrees of freedom are merely the difference between the degrees of freedom of the multiple group measurement models under the null and the alternative hypotheses. The Chi-square difference test results for testing the invariance of the factor loadings of the measurement model for political efficacy and political responsiveness are summarized in the MS-Excel workbook *EQFLS.XLS*. The contents of this file are shown in the following Microsoft Excel workbook.

The small P-value suggests that there is sufficient evidence that the null hypothesis should be rejected if a one percent level of significance is used. In other words, there is sufficient evidence that the factor loadings for the USA and Britain are different.

Invariance of factor loadings for political responsiveness
The measurement model in Figure 1 will now be used to illustrate how the multiple-group feature for ordinal variables of LISREL 8.80 for Windows may be used to assess whether or not the factor loadings for political responsiveness of the measurement model are invariant across the two countries. This assessment consists of testing the null hypothesis (H0) that the factor loadings for political responsiveness are identical across the two countries against the alternative hypothesis (H1) that the factor loadings for political responsiveness are not identical across the two countries. A Chi-square difference test is used to test H0 and H1.

The multiple-group model under H1 was already fitted to the data from the two countries in section 4. The multiple-group model under H0 may be fitted as follows.

Fitting the multiple group model under H0
➢ Use the *Open* option on the *File* menu to load the *Open* dialog box.
➢ Browse for the location *C:\LISREL 8.8 Student Examples\WORKSHOP.*
➢ Select the file *RH0MGCFA.SPL* by clicking on it.
➢ Click on the *Open* button to open the following text editor window.

```
RHOMGCFA.SPL

    Group 1: USA
    Observed Variables: NOSAY VOTING COMPLEX NOCARE TOUCH INTEREST
    Covariance Matrix from File USA.COV
    Asymptotic Covariance Matrix from File USA.ACM
    Sample Size: 1076
    Latent Variables: Efficacy Respons
    NOSAY =  1*Efficacy
    COMPLEX NOCARE = Efficacy
    TOUCH = 1*Respons
    INTEREST = Respons
    Group 2: BRITAIN
    Observed Variables: NOSAY VOTING COMPLEX NOCARE TOUCH INTEREST
    Covariance Matrix from File BRITAIN.COV
    Asymptotic Covariance Matrix from File BRITAIN.ACM
    Sample Size: 1266
    Latent Variables: Efficacy Respons
    NOSAY =  1*Efficacy
    COMPLEX NOCARE = Efficacy
    Set the Variance of Efficacy - Respons Free
    Set the Covariance between Efficacy and Respons
    Set the Error Variance of NOSAY Free
    Set the Error Variance of COMPLEX - INTEREST Free
    LISREL Output: ND=3 SC ME=WLS AD=OFF
    Path Diagram
    End of Problem
```

Line 1 provides a title for the USA group.
Line 2 specifies labels for the observed variables.
Lines 3 and 4 specify the matrix files for the polychoric covariance matrix and the estimated asymptotic covariance matrix of the polychoric variances and covariances respectively.
Line 5 specifies the number of observations in USA.PSF.
Line 6 specifies labels for the latent variables.
Lines 7 to 10 specify the measurement model for the USA group.
Line 11 provides a title for the Britain group.
Line 12 specifies labels for the observed variables.
Lines 13 and 14 specify the matrix files for the polychoric covariance matrix and the estimated asymptotic covariance matrix of the polychoric variances and covariances respectively.
Line 15 specifies the number of observations in BRITAIN.PSF.
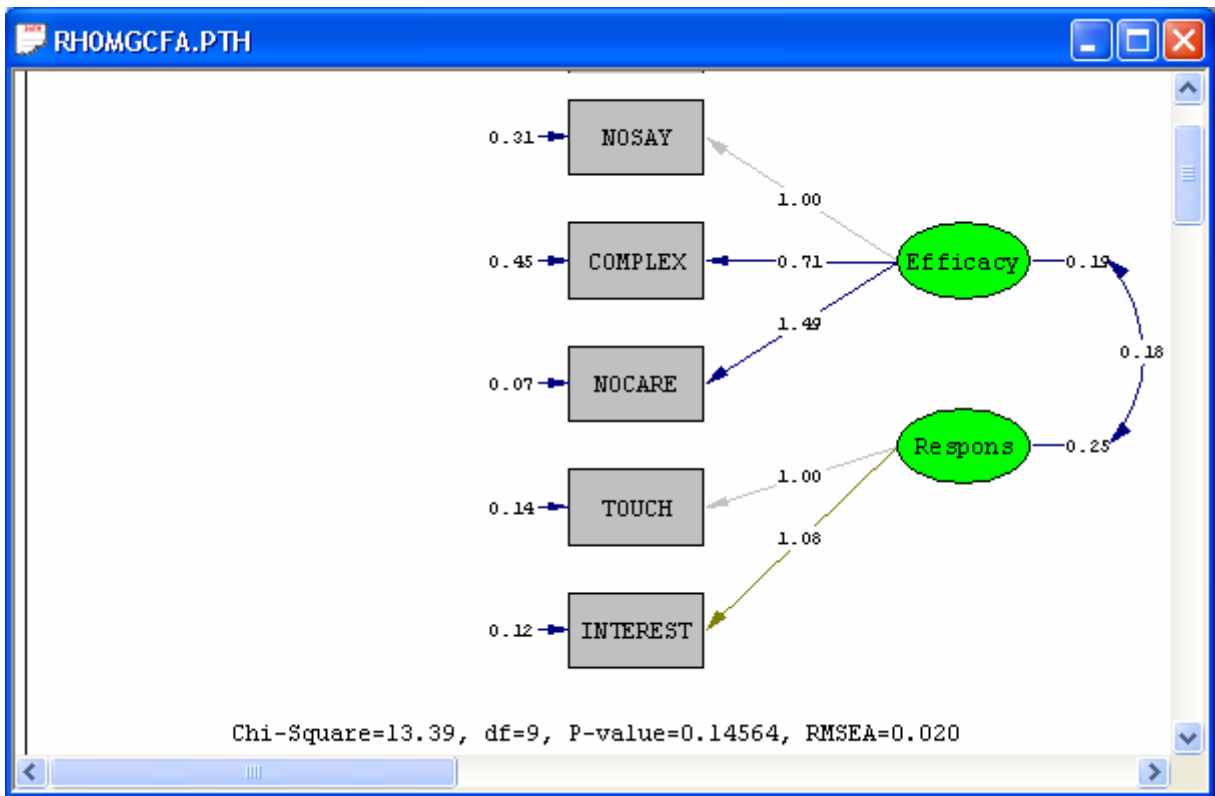Line 16 specifies labels for the latent variables.
Lines 17 to 22 specify the measurement model for the Britain group.
Line 23 specifies the Weighted Least Squares (WLS) method for ordinal variables and that the results should be printed in terms of the LISREL model for the measurement model in Figure 1.
Line 24 requests a path diagram.
Line 25 indicates no more SIMPLIS commands are to be processed.

➢ Click on the **Run LISREL** icon on the main toolbar to produce the following PTH window.

Multiple group variables for ordinal variables   11

Chi-Square=13.39, df=9, P-value=0.14564, RMSEA=0.020

Performing the Chi-square difference test

A Chi-square difference test is used to assess whether or not the factor loadings for political responsiveness of the measurement model are invariant across the two countries. In other words, a Chi-square difference test is used to test the null and alternative hypotheses. The test statistic value for the Chi-square difference test is merely the difference between the goodness-of-fit Chi-square test statistic values of the multiple group measurement models under the null and the alternative hypotheses. The associated degrees of freedom are merely the difference between the degrees of freedom of the multiple group measurement models under the null and the alternative hypotheses. The Chi-square difference test results for testing the invariance of the factor loadings for political responsiveness of the measurement model for political efficacy and political responsiveness are summarized in the MS-Excel workbook **EQRFLS.XLS**. The contents of this file are shown in the following Microsoft Excel workbook.

The large P-value suggests that there is insufficient evidence that the null hypothesis should be rejected. In other words, there is insufficient evidence that the factor loadings for political responsiveness of the USA and Britain are different.

References

Barnes, S.H. & Kaase, M. (Eds.). (1979).
   Political Action: Mass Participation in Five Western Democracies.
   Beverley Hills and London: Sage Publications.
Jöreskog, K.G. (1994).
   On the Estimation of Polychoric Correlations and their Asymptotic Covariance Matrix.
   Psychomatrika, **59**, 381-389.
Jöreskog, K.G. (2002).
   Structural Equation Modeling with Ordinal Variables using LISREL.
   Available at http://www.ssicentral.com/lisrel/ordinal.htm.
Jöreskog, K.G. & Sörbom, D. (1999).
   PRELIS 2: User's Reference Guide.
   Lincolnwood, IL: Scientific Software International, Inc.
Jöreskog, K.G. & Sörbom, D. (2006).
   LISREL 8.80 for Windows [Computer Software].
   Lincolnwood, IL: Scientific Software International, Inc.