

Structure Coefficients for the LISREL Model for Confirmatory Factor Analysis

by
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1. Introduction

Structure coefficients is a concept, which is often used in the research methodology in the Social Sciences, to refer to the elements of the estimated correlation matrix between the indicators and latent variables (factors) in measurement or confirmatory factor analysis models (see for example, Thompson (1997)). The Structural Equation Modeling (SEM) software package, AMOS (Arbuckle 1995) produces the structure coefficients for any structural equation model if the \$standardized and \$impliedmoments commands are specified. Although LISREL for Windows (Jöreskog & Sörbom, 2005) does not produce the structure coefficients directly, they can be computed from the LISREL output file.

This paper describes a procedure to compute the structure coefficients for a confirmatory factor analysis model from the corresponding LISREL results. The LISREL submodel for confirmatory factor analysis is formulated in section 2. The formula for the estimated correlation matrix of the indicators and latent variables (factors) of the LISREL submodel of section 2 is provided in section 3. An illustrative example is provided in section 4.

2. The LISREL Submodel for Confirmatory Factor Analysis

The Confirmatory Factor Analysis LISREL submodel (Jöreskog 1973) may be expressed as

$$\mathbf{x} = \mathbf{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta} \quad (1)$$

where \mathbf{x} denotes a $p \times 1$ vector of indicators of the m exogenous latent variables $\boldsymbol{\xi}$, $\mathbf{\Lambda}_x$ denotes the corresponding $p \times m$ matrix of unknown factor loadings, $\boldsymbol{\delta}$ denotes a $p \times 1$ vector of measurement errors. From (1), the structural model for the population covariance matrix, $\boldsymbol{\Sigma}$, follows as

$$\boldsymbol{\Sigma} = \mathbf{\Lambda}_x \boldsymbol{\Phi}_\xi \mathbf{\Lambda}_x' + \boldsymbol{\Theta}_\delta \quad (2)$$

where $\boldsymbol{\Phi}_\xi$ and $\boldsymbol{\Theta}_\delta$ denote the covariance matrices of $\boldsymbol{\xi}$ and $\boldsymbol{\delta}$ respectively.

3. The Structure Coefficients

Suppose that the $t \times 1$ vector z consists of all the variables of the LISREL submodel in (1), i.e.

$$z = \begin{bmatrix} x \\ \xi \\ \delta \end{bmatrix} \quad (3)$$

Suppose that the elements of the $t \times t$ matrix B denote the regression weights of the relationships between all the variables of the LISREL model, i.e.

$$z = Bz + z_e \quad (4)$$

where

$$B = \begin{bmatrix} 0 & \Lambda_x & I_{p_x} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (5)$$

where I_n denotes the $n \times n$ identity matrix and

$$z_e = \begin{bmatrix} 0 \\ \xi \\ \delta \end{bmatrix} \quad (6)$$

Suppose that Φ denote the covariance matrix of z_e , i.e.

$$\Phi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Phi_\xi & 0 \\ 0 & 0 & \Theta_\delta \end{bmatrix} \quad (7)$$

By using expressions (4) to (7), the $t \times t$ covariance matrix, Υ_t , of z may be expressed as

$$\Upsilon_t = (I_t - B)^{-1} \Phi (I_t - B)^{-1'} = \Lambda_t \Phi \Lambda_t' \quad (8)$$

where $\Lambda_t = (I_t - B)^{-1}$. From (8), the $(p+m) \times (p+m)$ covariance matrix, Υ , of the p observed and m latent variables follows as

$$\Upsilon = [I_{p+m} \quad 0] (I_t - B)^{-1} \Phi (I_t - B)^{-1'} [I_{p+m} \quad 0]' = \Lambda \Phi \Lambda' \quad (9)$$

where Λ denotes the $(p+m) \times t$ matrix consisting of the first $p+m$ rows of Λ_t . From expression (9), it follows by using expressions (4) to (7) that

$$\Upsilon = \begin{bmatrix} \Lambda_x \Phi_\xi \Lambda_x' + \Theta_\delta & \Lambda_x \Phi_\xi \\ \Phi_\xi \Lambda_x' & \Phi_\xi \end{bmatrix} \quad (10)$$

From (10), it follows that the correlation matrix of the indicators and the latent variables may be expressed as

$$\Psi = \begin{bmatrix} D_\sigma^{-1} [\Lambda_x \Phi_\xi \Lambda_x' + \Theta_\delta] D_\sigma^{-1} & D_\sigma^{-1} \Lambda_x \Phi_\xi D_\phi^{-1} \\ D_\phi^{-1} \Phi_\xi \Lambda_x' D_\sigma^{-1} & D_\phi^{-1} \Phi_\xi D_\phi^{-1} \end{bmatrix} \quad (11)$$

where D_σ and D_ϕ denote diagonal matrices with the standard deviations of the indicators and the latent variables on the diagonal respectively. From (11), the structure coefficients follow as the elements of the matrix

$$\hat{\Psi}_s = \hat{D}_\sigma^{-1} \hat{\Lambda}_x \hat{\Phi}_\xi \hat{D}_\phi^{-1} \quad (12)$$

4. An Illustrative Example

Graham, Guthrie & Thompson (2003) provides the factor pattern and structure coefficients for a confirmatory factor analysis model for six indicators and two latent variables fitted to a heuristic covariance matrix. We will use this example to illustrate how the results in the LISREL output file may be used to obtain the factor pattern and structure coefficients reported by Graham, Guthrie & Thompson (2003, Table 2 on p. 147).

The following LISREL syntax file may be used to obtain the factor pattern coefficients.

The LISREL Syntax File

```
DA NI=6 NO=100 MA=CM
LA
A B C D E F
CM
2.497
3.007 9.990
3.487 6.014 9.990
2.537 4.446 4.545 9.990
2.188 3.836 3.926 6.873 9.990
2.537 4.446 4.545 7.023 6.164 9.990
M0 NX=6 NK=2 LX=FU,FI PH=ST,FR TD=DI,FR
FR LX(1,1) LX(2,1) LX(3,1) LX(4,2) LX(5,2) LX(6,2)
PD
OU ND=4 SC
```

Note that the SC option is specified to obtain the Completely Standardized solution. If the syntax file above is processed by LISREL for Windows, the matrix of pattern coefficients is listed as the matrix LAMBDA_X in the Completely Standardized Solution section in the LISREL output file. This matrix is given by

LAMBDA-X		
	KSI 1	KSI 2
	-----	-----
A	0.8493	- -
B	0.7260	- -
C	0.8167	- -
D	- -	0.8752
E	- -	0.7737
F	- -	0.8085

The factor correlation matrix is listed as the matrix PHI in the Completely Standardized Solution section in the LISREL output file. This matrix is given by

PHI		
	KSI 1	KSI 2
	-----	-----
KSI 1	1.0000	
KSI 2	0.6824	1.0000

The structure coefficient matrix in (12) is now obtained as the matrix product of LAMBDA_X and PHI above and follows as

$$\hat{\Psi}_s = \begin{bmatrix} 0.8493 & 0 \\ 0.7260 & 0 \\ 0.8167 & 0 \\ 0 & 0.8752 \\ 0 & 0.7737 \\ 0 & 0.8085 \end{bmatrix} \begin{bmatrix} 1 & 0.6824 \\ 0.6824 & 1 \end{bmatrix} = \begin{bmatrix} 0.849 & 0.580 \\ 0.726 & 0.495 \\ 0.817 & 0.557 \\ 0.597 & 0.875 \\ 0.528 & 0.774 \\ 0.552 & 0.808 \end{bmatrix} \quad (13)$$

The structure coefficients in (13) agree up to 3 decimal places with those reported by Graham, Guthrie & Thompson (2003, Table 2 on p. 147).

Instead of using the LISREL syntax file, the following SIMPLIS syntax file may be used to compute the structure coefficients as illustrated here.

The SIMPLIS Syntax File

```
Observed Variables
A B C D E F
Latent Variables
FACTOR1 FACTOR2
Covariance Matrix
2.497
3.007 9.990
3.487 6.014 9.990
2.537 4.446 4.545 9.990
2.188 3.836 3.926 6.873 9.990
2.537 4.446 4.545 7.023 6.164 9.990
Sample Size = 100
Relationships
A - C = FACTOR1
D - F = FACTOR2
LISREL Output: ND=4 SC
Path Diagram
End of Problem
```

References

- Arbuckle, J.L. (1995). *Amos 3.5 for Windows*. Chicago: SmallWaters Corporation.
- Graham, J.M., Guthrie, A.C. & Thompson, B. (2003). Consequences of Not Interpreting Structure Coefficients in Published CFA Research: A Reminder. *Structural Equation Modeling*, **10**, 142-153.
- Jöreskog, K.G. (1973). A General Method for Estimating a Linear Structural Equation System. In: A.S. Goldberger & O.D. Duncan (Eds.), *Structural Equation Models in the Social Sciences*. New York: Seminar Press.
- Jöreskog, K.G. & Sörbom, D. (2005). *LISREL for Windows [Computer Software]*. Lincolnwood, IL: Scientific Software International, Inc.
- Thompson, B. (1997). The importance of structure coefficients in structural equation modeling confirmatory factor analysis. *Educational and Psychological Measurement*, **57**, 5-19.